Bisect an angle

Given an angle construct a line that bisects the angle.

Proof: Let \( \angle ABC \) be the given angle.

Draw a circle with center at \( B \) and radius shorter (or equal) to both \( AB \) and \( BC \). Let \( D \) and \( E \) be the intersections of this circle with \( AB \) and \( BC \). Draw circles with centers at \( D \) and \( E \) that have radius greater than \( DE \).

Let \( F \) be the intersection of these last two circles.

Claim: \( \angle BF \) bisects \( \angle ABC \).

Consider triangles \( \triangle BDF \) and \( \triangle BEF \). Note that \( BD = BE \) since both are radii of some circle. Note that \( DF = DE \) because the circles centered at \( D \) and \( E \) have the same radius. Of course \( BF = BF \) so \( \triangle BDF \cong \triangle BEF \) by SSS. Therefore \( \angle BDF = \angle EBF \).