8.5 Probability

Example 1. A typical jet engine lasts at most 10 years (after that it is replaced automatically). Let $p(t)$ be the probability density function describing $t$, the total life span of a typical engine. The graph of $p(t)$ is given below, where $C$ is unknown.

(a) What is the value of $C$?
(b) What is the probability that the jet engine breaks in its first year?
(c) What is the probability that the jet engine breaks in its 10th year?
(d) What is the probability that the jet engine lasts at least 5 years?
Example 2. [Stewart, 6e, 8.5#10a] A type of light bulb is labeled as having an average lifetime of 1000 hours. It’s reasonable to model the probability of failure of these bulbs by an exponential density function with mean $\mu = 1000$. Use this model to find the probability that a bulb

(a) fails within the first 200 hours
(b) burns for more than 800 hours
Example 3. Find the median of the light bulbs described in Example 2.

Example 4. [Stewart, 6e, 8.5#12] According to the National Health Survey, the heights of adult males in the United States are normally distributed with mean 69.0 inches and standard deviation 2.8 inches.

(a) What is the probability that an adult male chosen at random is between 65 inches and 73 inches tall?
(b) What percentage of the adult male population is more than 6 feet tall?
If we integrate $\int_{73}^{65} f(x) \, dx$ we need to use our calculator to do this: there is no basic formula for the antiderivative. We can use the general calculator integrator or the calculator's built-in normal distribution function. In either case we should get $\int_{73}^{65} f(x) \, dx \approx r$. 

If you repeat the calculation with $\int_{\infty}^{72} f(x) \, dx$ you just use a large value such as $\infty$ or $\pi$. You should get $\int_{\infty}^{72} f(x) \, dx \approx r$. 

This shows that either $\infty$ or $\pi$ are reasonably close approximations of $\infty$ for this problem.