Example 1. [Integrals from Stewart] For each of the following integrals, indicate what technique should be used, at least as a starting point. Your answer should be one or more of the following: BA, AS/R, US, etc. Perhaps some work will need to be done to figure out the strategy, but in all cases stop before doing any integral of any kind. Also stop before doing any partial fraction or polynomial division.

**BA** Basic anti-derivatives

**AS/R** Algebraic simplification or rewriting.

**US** \( u \)-substitution.

**TP** Trigonometric powers.

**IBP** Integration by parts.

**PF** Partial fractions.

**PD** Polynomial division.

**CS** Completing the square.

**TS** Trigonometric substitution.

**RS** Rationalizing substitution.

**NOA** None of the above.

(a) \( \int \frac{\tan^3(x)}{\cos^3(x)} \, dx \)

(b) \( \int e^{\sqrt{x}} \, dx \)

(c) \( \int \frac{x^5 + 1}{x^3 - 3x^2 - 10x} \, dx \)

(d) \( \int \frac{dx}{x\sqrt{\ln(x)}} \)

(e) \( \int \sqrt{\frac{1-x}{1+x}} \, dx \)

(f) \( \int e^2 \, dx \)

(g) \( \int \frac{x - 1}{x^2 - 4x + 5} \, dx \)

(h) \( \int \frac{x^2}{1 - x^2} \, dx \)

(i) \( \int \sqrt{\frac{2x - 1}{2x + 3}} \, dx \)

(j) \( \int \frac{\sin(x) + \sec(x)}{\tan(x)} \, dx \)

(k) \( \int x^8 \sin(x) \, dx \)

(l) \( \int \sqrt{3 - 2x - x^2} \, dx \)

(m) \( \int \frac{x - 1}{x^2 - 4x - 5} \, dx \)

(n) \( \int \frac{x}{1 - x^2 + \sqrt{1 - x^2}} \, dx \)
This gives us \( \sqrt{m - x} \cdot \sqrt{m - x} = m - x \cdot \sqrt{m - x^2} \). Note that the first integral is sin \(-1\) dx and that the second is a simple \( u \) substitution \( u = m - x^2 \). Specifically, note that 2 is just a constant number. Note that we can't easily factor the bottom; hence, we have to start with this since the degree of the top equals the degree of the bottom. Note that once we do polynomial division, the remainder becomes \( \star \). If “\( \star \)” is a constant or an \( x \) or a constant plus \( x \) how can we still have an integral from Table ??

Our integral becomes

\[
\int \sqrt{n x - m} \, dx - \int \frac{x}{\sqrt{n x - m}} \, dx
\]

To get rid of all the 's solve for backwards substitution

\[
u = \sqrt{n x - m} \Rightarrow x = \frac{m}{n} \Rightarrow du = \frac{m}{n} \, dx
\]

Now apply PD, specifically sin \( x \) and tan \( x \)

cos \( x \) and tan \( x \)

= \( \cos x \)

IBP actually probably tabular integration by parts; here we get

\[
\int - \frac{1}{u^2} \, du \cdot \frac{1}{u^2} \, du = \int u \, du
\]

Now apply PD followed by Table R, specifically

d \( x \) and Table S. Factor out the negative from in front of \( x^2 \) like so

\[
\int - \frac{dx}{x^2} \cdot \frac{m}{x^2} = \int \frac{m}{x^2} \, dx
\]

= \( \frac{m}{x} \)

PF and ASkR followed by TP, specifically

d \( x \) and Table S. Here are some details

\[
u = \sqrt{m - x^2} \Rightarrow x = m \Rightarrow du = \frac{m}{n} \, dx
\]

But they all amount to the same thing

\[
v = \frac{m}{n}
\]

PF and ASkR and US. There are different ways you can do this one, but they all amount to the same thing.
\[ \int_{m-x^2}^x g(u) \, du = -\int_{m-x^2}^x u \, du = -\int_{m-x^2}^x u^2 \, du \]