

7.2 Trigonometric Integrals

Rule. To integrate $\int \sin^n(x) \cos^m(x) \, dx$:

Case 1 If the power of sine is odd, then let $u = \cos(x)$, $du = -\sin(x) \, dx$, and get rid of all but one power of $\sin(x)$ using

\[
\begin{align*}
\sin^2(x) &= 1 - \cos^2(x) \\
\sin^4(x) &= (1 - \cos^2(x))^2 \\
\sin^6(x) &= (1 - \cos^2(x))^3 \\
\text{etc}
\end{align*}
\]

and then complete your $u$-substitution and integrate.

Case 2 If the power of cosine is odd, then let $u = \sin(x)$, $du = \cos(x) \, dx$, and get rid of all but one power of $\cos(x)$ using

\[
\begin{align*}
\cos^2(x) &= 1 - \sin^2(x) \\
\cos^4(x) &= (1 - \sin^2(x))^2 \\
\cos^6(x) &= (1 - \sin^2(x))^3 \\
\text{etc}
\end{align*}
\]

and then complete your $u$-substitution and integrate.

Case 3 If both sine and cosine have even powers, then use the identities

\[
\begin{align*}
\sin^2(\theta) &= \frac{1}{2} (1 - \cos(2\theta)) \\
\cos^2(\theta) &= \frac{1}{2} (1 + \cos(2\theta))
\end{align*}
\]

then multiply everything out. You now have only powers of $\cos(2x)$. Split the integral up: odd powers bigger than 1, go to Case 2; even powers, repeat Case 3. In this way you are eventually left with only single powers of $\cos(2x)$, $\cos(4x)$, $\cos(8x)$, $\ldots$, which you can finish immediately.

Rule. To integrate $\int \tan^n(x) \sec^m(x) \, dx$

Case 1 If the power of tangent is odd get then let $u = \sec(x)$, $du = \sec(x) \tan(x) \, dx$, and get rid of all but one power of $\tan(x)$ using

\[
\begin{align*}
\tan^2(x) &= \sec^2(x) - 1 \\
\tan^4(x) &= (\sec^2(x) - 1)^2 \\
\tan^6(x) &= (\sec^2(x) - 1)^3 \\
\text{etc}
\end{align*}
\]

and then complete your $u$-substitution and integrate.

Case 2 If the power of secant is even then let $u = \tan(x)$, $du = \sec^2(x) \, dx$, and get rid of all but two powers of $\sec(x)$ using

\[
\sec^2(x) = \tan^2(x) + 1
\]
sec^4(x) = (\tan^2(x) + 1)^2 \\
sec^6(x) = (\tan^2(x) + 1)^3 \\
etc

and then complete your u-substitution and integrate.

**Case 3** If tangent has an even power and secant an odd power, then get rid of all the powers of \( \tan(x) \) using \( \tan^2(x) = \sec^2(x) - 1 \) as above. Now we have only powers of \( \sec(x) \). Use a little luck, integration by parts, the secant-tangent identity, and the following:

\[
\int \sec(x) \, dx = \ln |\sec(x) + \tan(x)|.
\]