11.8 Power Series.

Example 1. Find the Radius of Convergence, and draw the resulting interval
\[ f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x - 1/2)^n}{n} \]

Solution.
\[
L = \lim_{n \to \infty} \left| \frac{(-1)^{n+1}(x - 1/2)^{n+1}}{n+1} \cdot \frac{(-1)^n(x - 1/2)^n}{n} \right|
\]
\[
= \lim_{n \to \infty} \left| \frac{(-1)^{2n}}{(n+1)n} \cdot \frac{(x - 1/2)^2}{1} \right|
\]
\[
= \lim_{n \to \infty} \left| \frac{1}{n(n+1)} \cdot (x - 1/2)^2 \right|
\]
\[
= |x - 1/2|^2
\]
The best way to understand \(|x - 1/2| < 1\) is in terms of distances. This inequality is equivalent to the question: “which \(x\)’s have a distance < 1 from 1/2?".

Example 2. Find the Radius of convergence and draw the resulting interval
\[
\sum_{n=0}^{\infty} (2x - 3)^n
\]

One algebraic rule for absolute values states that if \(a > 0\), then \(a|x + b| = |ax + ab|\).
Example 3. Find the Radius of Convergence and draw the resulting interval

\[
\sum_{n=0}^{\infty} \frac{(x - 7)^n}{11^n}
\]

So \(L = \frac{1}{11}\), we need this to be < 1, so

\[
\frac{1}{11} |x - 7| < 1
\]

From this we see that \(R = 1\).