11.7 Strategy for Series

Here’s a list of tests which you might use. Tests marked with (*) require 100% positive terms. The test marked with (**) requires 100% alternating terms.

- **GST** Geometric Series Test
- **DT** Divergence Test
- (***) **IT** Integral Test
- (*) **PST** $p$-Series Test
- (*) **ICT** Inequality Comparison Test
- (*) **LCT** Limit Comparison Test
- (***) **AST** Alternating Series Test
- **ACT** Absolute Convergence Test
- **RAT** Ratio test
- **ROT** Root test

- **•** Size the series up: identify the dominant parts. If \( \lim_{n \to \infty} c_n \neq 0 \), then apply the divergence test.

- **•** **Ratio test.** Good general fallback.
  1. Almost always works if series has factorials (like \( n! \) or \((2n + 1)!)\) and/or geometric factors (like \( 3^n \) or \( \frac{1}{(-1)^n} \)).
  2. Never works if series has only powers of \( n \) (like \( n^2, \sqrt{n + 1}, \ldots \)) and/or powers of \( \ln(n) \) (like \( \sqrt{\ln(n)} \) or \((\ln(n))^2\)).

- **•** **Special forms.**
  1. \( p \)-series
  2. geometric series
  3. alternating series

- **•** **Other forms**
  1. **Integral test.** If you can integrate the function.
  2. **Limit comparison tests.** If you have only powers of \( n \) (like \( n, \frac{1}{n}, \sqrt{n^2 + 1}, \text{etc.} \)).
  3. **Inequality comparison test.** If part of the series is a function with a built-in inequality (like \(-1 \leq \sin(x), \cos(x) \leq 1 \) or \(-\pi/2 \leq \tan^{-1}(x) \leq \pi/2\)).
  4. **Absolute convergence test.** When you want to apply one of the other tests (like **IT**, **PST**, **ICT**, **LCT**) but first need to make sure the terms are positive.
  5. **Root test.** If the series has \( \star^n \) where \( \star \) is some big complicated formula.

- **•** **Conditional convergence.** A combination of two previous tests: probably the **AST** for convergence, and then something else to show \( \sum |c| \) diverges (note: the something else cannot be **RAT** or **ROT** or **GST** or **DT**: that leaves **PST**, **IT**, **ICT**, **LCT**).