Chapter 11. Sequences and Series

11.5 Alternating Series

Example 1. [Alternating Harmonic Series] Show that \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \] converges.

Solution. We check the conditions:

1. \[ \frac{(-1)^{n+1}}{n+1} \leq \frac{(-1)^{n}}{n} \]
   since this is equivalent to saying \[ n \geq 1 \] for all \[ n \geq 3 \].

2. \[ \lim_{n \to \infty} \frac{(-1)^{n+1}}{n} = 0 \]
   since \[ \lim_{n \to \infty} \frac{\ln(n)}{n} = 0 \].

Therefore, by the Alternating Series Test, \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \] converges.

Example 2. Show that the series \[ \sum_{n=1}^{\infty} \frac{(-1)^{n} \ln(n)}{n} \] converges.