10.1 Curves Defined by Parametric Equations

**Definition.** Let \((x, y)\) be a point on a curve. Suppose that \(x\) and \(y\) are each given as a function of a third variable,

\[
x = f(t), \ y = g(t)
\]

then we call these equations **parametric equations** and call \(t\) the **parameter**.

10.2 Calculus with Parametric Curves

**Fact.** Given parametric equations \(x = f(t)\) and \(y = g(t)\), the Cartesian derivative equals

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}.
\]

To understand this, recall that

\[
\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} = \frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}
\]