Your name: __________________________

Your student number: __________________________

Showing work. Each answer must include step-by-step work on your page whenever possible (so one exception would be #1). Even calculations which will be done in a calculator, should first appear as formulas on the page (here, “formulas” can mean function expressions, like $C(50) - C(49)$ or $f(100) + \Delta x$ or $-4(5^3) - e^5$). All critical points and inflection points should be found algebraically.

Time management. Probably not everyone will have enough time to do every problem correctly. I think it is better, and that you will get a better score, if you (1) skip the hardest problems until later, (2) work carefully and write more complete steps so that you don’t make mistakes and you get better partial credit, and you can see which part of your work is correct, etc.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible points</th>
<th>Points received</th>
</tr>
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Please sign the following pledge:

On my honor I have neither given nor received any aid on this exam; I have upheld the ideals of the honor code.

Signature __________________________

Percentage 96
1. Assume that the following is a graph of \( f(x) \).

List all the critical points of \( f(x) \), and identify each as a local max/min/neither.

2. Let \( f(x) = 3x^5 - 5x^3 \).

   A plot of \( f(x) \) is shown below (but somehow my computer seems to be broken: it doesn’t show any of the axes, or numbers or tickmarks. Sorry.)

   (a) Find the critical points of \( f(x) \).
   (b) Make a “1D# table” (1st Derivative Number Line Table) that shows the first derivative test and the conclusions that it gives you.
   (c) List the intervals of increase/decrease/concavity
   (d) Identify any inflection points

3. Let \( f(x) = \frac{8}{3}x^3 - 36x^2 + 10000 \).
(a) Find the critical points of \( f(x) \).

(b) Apply the second derivative test to each of the the critical points and identify each critical point as a local max/min/neither. Be sure to write down the steps you use to apply the second derivative test.

4. Find the inflection points of \( f(x) = x^5 - 5x^4 + 35 \).

5. Find the critical points of

(a) \( f(x) = x^2e^x \)

(b) \( f(x) = \frac{1}{2}x^2 - 3x + 2 \ln(x) \)

6. Let \( f(x) = x^2 \ln(x) - 10x \ln(x) + 5x \). Suppose that the only critical points are \( x = e^{-1/2} \) and \( x = 5 \). Use the Global Max/Min test to find the Global maximum and minimum (both \( x \)-value and \( y \)-value) on the interval \( 0.1 \leq x \leq 10 \). (Note: double and triple check that you’ve entered your function in the calculator correctly.)

7. Shown below are a cost and a revenue curve. Estimate the production level that maximizes profit and estimate the profit at that point.

![Graph of Cost and Revenue](image)

8. Suppose that your T-shirt company makes a revenue of \( R(q) = 10q \) and has a cost of \( C(q) = 0.003q^3 + 5.6q + 3.4 \). At what quantity is profit maximized? (Note: find this value algebraically, but don’t bother applying a test to make sure that the \( q \) you find is a maximum as opposed to a minimum.)

9. Shown below is the graph of a total cost function of making \( q \) items. Use the graph to answer the following questions:

(a) Find the total cost of making 50 items.

(b) Estimate the marginal cost when \( q = 50 \); give units.

(c) Estimate the marginal cost when \( q = 100 \).

(d) Estimate the total cost for \( q = 120 \).
10. Suppose a manufacturer is making 3000 units of some item. They are selling the item for $15 per unit, the marginal cost is $22, and the total cost is $41,111. If we increase production above 3000, and assuming that we sell all the units we produce, which of the following would increase? Which would decrease? Which would be impossible to tell? Why?

(a) Revenue.
(b) Profit.
(c) Total cost.
(d) Marginal cost.
(e) Marginal revenue.
(f) Average cost.
(g) Elasticity.

11. A company is making complicated things with a cost function given by $C(q) = 7e^{-q} - 5\sqrt{q} + 11000q^2$ and revenue function given by $R(q) = 25000q$.

(a) Find the marginal cost and marginal revenue at $q = 300$.
(b) At $q = 300$ is the profit increasing or decreasing? How does your answer to (a) tell you this?

12. The table below shows marginal revenue and marginal cost at various points.

<table>
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<tr>
<th>$q$</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
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<tbody>
<tr>
<td>MR</td>
<td>2.00</td>
<td>1.25</td>
<td>1.05</td>
<td>0.95</td>
<td>0.89</td>
<td>0.84</td>
<td>0.80</td>
<td>0.77</td>
<td>0.75</td>
<td>0.72</td>
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<tr>
<td>MC</td>
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<td>1.35</td>
<td>0.78</td>
<td>0.39</td>
<td>0.18</td>
<td>0.15</td>
<td>0.30</td>
<td>0.63</td>
<td>1.14</td>
<td>1.83</td>
<td>2.70</td>
</tr>
</tbody>
</table>

(a) If the production level is $q = 400$, should the company make a small increase in production? Why or why not?
(b) Find two critical points of profit. Which one has maximum profit? Why?

13. The graph below shows the total cost of T-shirt manufacturing for your company. Estimate the lowest average cost, both the $q$-value and the $a(q)$-value.
14. A company is making simple things with a cost function given

<table>
<thead>
<tr>
<th>$q$</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(q)$</td>
<td>1.00</td>
<td>2.25</td>
<td>6.00</td>
<td>12.25</td>
<td>21.00</td>
<td>32.25</td>
<td>46.00</td>
</tr>
<tr>
<td>$MC$</td>
<td>0.00</td>
<td>5.00</td>
<td>10.00</td>
<td>15.00</td>
<td>20.00</td>
<td>25.00</td>
<td>30.00</td>
</tr>
</tbody>
</table>

(a) Find the average cost at $q = 3$.
(b) Is the average cost increasing or decreasing at $q = 3$?

15. The elasticity of gasoline is $E = 0.26$. In the summer, gas prices increase by 10%. What is the expected change in demand for gasoline?

16. Suppose T-shirts have a demand function of $q = 10 + \frac{1000}{p + 1}$.

(a) Find the exact elasticity of T-shirts at $p = 11$.
(b) Based on your answer to (a), will increasing the price cause the revenue to increase or decrease?

17. You are given a price and demand from the following table

<table>
<thead>
<tr>
<th>$p$</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
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<th>4.5</th>
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<tbody>
<tr>
<td>$q$</td>
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<td>990</td>
<td>978</td>
<td>960</td>
<td>938</td>
<td>910</td>
<td>878</td>
<td>840</td>
<td>798</td>
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(a) Find the elasticity at $p = 5$.
(b) If we increase price, do we expect $R$ to increase or decrease? Why?
(c) If we increase price 7%, how much of a percentage change do we expect from demand?

18. An object is falling with velocity given by $v(t) = -9.8t + 12$, from $t = 2$ to $t = 5$ (where $t$ is in seconds, and $v$ is in meters/second).

(a) Write a Right Hand Riemann Sum using $n = 6$ that approximates the distance travelled.
(Note: “Write” means write. Do not calculate anything at all, do no multiplication or addition. Just write down the formula, with everything filled in, so that you could give it to someone who knew no Calculus, but had a 4 function calculator, and they’d be able to calculate the answer.)
(b) Calculate the Riemann Sum in part (a)

(c) Write an integral that would give the exact value for the distance travelled by the object from $t = 2$ to $t = 5$. 