Things to keep in mind as you take this practice test:

- The real test will not be this long. It will probably have around 8 problems.
- I tend to avoid harder problems on the test, but I don’t avoid them much on the practice.
- You should know/memorize/write down the following:
  - the rules for exponents and adding, subtracting multiplying and dividing fractions,
  - the $x$ and $y$ intercepts of straight lines, and also $e^x$, $\ln(x)$,
  - how to solve equations that involve $e^x$ and $\ln(x)$.
- Since this test is for practice you should think about doing variations of some of the problems, especially the ones that you find difficult.
- Everything should be done algebraically unless explicitly stated otherwise, or where it is not applicable, like a problem involving only the picture of a graph or a table of numbers. On the real midterm, whenever it’s possible, I will require you to write algebraic steps that lead up to your answer, even in problems that involve a calculator.
Showing work. Each answer must include step-by-step work on your page (except for # ??). Even calculations which will be done in a calculator, should first appear as formulas on the page (here, “formulas” can mean function expressions, like $C(50) - C(49)$ or $f(100) + \Delta x$ or $-4(5^3) - e^5$).

Time management. Probably not everyone will have enough time to do every problem correctly. I think it is better, and that you will get a better score, if you (1) skip the hardest problems until later, (2) work carefully and write more complete steps so that you don’t make mistakes and you get better partial credit, and you can see which part of your work is correct, etc.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible points</th>
<th>Points received</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
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<tr>
<td>5</td>
<td>12</td>
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<tr>
<td>6</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

Please sign the following pledge:

On my honor I have neither given nor received any aid on this exam; I have upheld the ideals of the honor code.

Signature ____________________________
1. This problem refers to the graph below.
   (a) Find $f(1), f(-0.5)$.
   (b) Solve $f(x) = 2$.

![Graph](image)

2. This problem refers to the table below.
   (a) Find $f(1), f(-0.5)$.
   (b) Solve $f(x) = 2$ (your solution(s) will be approximations, and you can assume that $f(x)$ is continuous and defined for all values)

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2.50</th>
<th>-2.00</th>
<th>-1.50</th>
<th>-1.00</th>
<th>-0.50</th>
<th>0.00</th>
<th>0.50</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0.02</td>
<td>0.18</td>
<td>1.05</td>
<td>3.68</td>
<td>7.79</td>
<td>10.00</td>
<td>7.79</td>
<td>3.68</td>
<td>1.05</td>
<td>0.18</td>
<td>0.02</td>
</tr>
</tbody>
</table>

3. This problem refers to the graph below. On what intervals is the graph increasing? On what intervals is it decreasing?

![Graph](image)

4. This problem refers to the table below. On what intervals is $f(x)$ increasing? On what intervals is it decreasing?

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3.00</th>
<th>-2.50</th>
<th>-2.00</th>
<th>-1.50</th>
<th>-1.00</th>
<th>-0.50</th>
<th>0.00</th>
<th>0.50</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-0.38</td>
<td>-0.13</td>
<td>0.44</td>
<td>0.65</td>
<td>0.00</td>
<td>-0.88</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
<td>-1.21</td>
<td>-1.02</td>
<td>0.38</td>
<td>1.53</td>
</tr>
</tbody>
</table>
5. Solve \( f(x) = 3 \) where \( f(x) = x^2 - 1 \).

6. Find the equation of the line through the points \((3, 7)\) and \((-1, -10)\).

7. (Hughes-Hallett, 4e, 1.2#13) A company rents cars at $40 a day and 15 cents a mile. It's competitor's cars are $50 a day and 10 cents a mile.
   
   (a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.
   
   (b) On the same axes, graph both functions.

   (c) How should you decide which company is cheaper?

8. (Hughes-Hallett, 4e, 1.3#14) When a deposit of $1000 is made into an account paying 8% interest, compounded annually, the balance, $\(B\), in the account after \(t\) years is given by \(B = 1000(1.08)^t\). Find the average rate of change in the balance over the interval \(t = 0\) to \(t = 5\). Give units and interpret your answer in terms of the balance in the account.

9. Suppose a falling rock has position given by the following formula:
   
   \[ p(t) = -4.9t^2 + 13t + 10, \]
   
   where \(p\) is measured in meters and \(t\) in seconds. Find the average velocity from \(t = 1\) to \(t = 2\) of the rock, including units.

10. Suppose an object is moving with position given in the table below. falling rock has position given by the following table where \(p\) is measured in meters and \(t\) in seconds. Find the average velocity from \(t = 1\) to \(t = 2\) of the rock, including units.

<table>
<thead>
<tr>
<th>(t)</th>
<th>0.00</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.25</th>
<th>2.50</th>
<th>2.75</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p(t))</td>
<td>10.00</td>
<td>12.94</td>
<td>15.28</td>
<td>16.99</td>
<td>18.10</td>
<td>18.59</td>
<td>18.48</td>
<td>17.74</td>
<td>16.40</td>
<td>14.44</td>
<td>11.88</td>
<td>8.69</td>
<td>4.90</td>
</tr>
</tbody>
</table>

11. A company is going to make a new kind of glue. To set up the factor, pay for the building, buy the machines, etc. will cost $1,225,000. Each tube of glue will cost $0.50 to make. They will sell each tube for $2.

   (a) Find a formula for the cost function, the revenue function, and the profit function.

   (b) Find the break even point.

12. A company is making all electric sports cars. Their cost function is \(C(q) = 10 + 1.5q\) where \(q\) is the number of cars they make and \(C\) is measure in millions of dollars.

   (a) Suppose the company can sell 1 car if the price it at \(p = 0.5\) (i.e. half a million dollars), and they can sell 10 cars if they price it at \(p = 0.1\) (i.e. $100,000). Assume that demand is linear and write a formula for the demand function.

   (b) Combine your answer to part (a) with the cost function to have a formula for \(C(p)\), i.e. cost as a function of price.

13. (Hughes-Hallett, 4e, 1.4#28) The demand curve for a product is given by \(q = 120,000 - 500p\) and the supply curve is given by \(q = 1000p\) for \(0 \leq p \leq 240\), where price is in dollars.

   (a) At a price of $100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?
(b) Find the equilibrium price and quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to the equilibrium?

14. Below we show two “curves”, i.e. examples of \( q \) as a function of \( p \). In each case “\( k \)” is short for “thousand”, so 120\( k \) is 120000. One of the functions is supply and one is demand.

<table>
<thead>
<tr>
<th>( p )</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
<th>220</th>
<th>240</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q = \text{curve 1} )</td>
<td>120( k )</td>
<td>110( k )</td>
<td>100( k )</td>
<td>90( k )</td>
<td>80( k )</td>
<td>70( k )</td>
<td>60( k )</td>
<td>50( k )</td>
<td>40( k )</td>
<td>30( k )</td>
<td>20( k )</td>
<td>10( k )</td>
<td>0( k )</td>
</tr>
<tr>
<td>( q = \text{curve 2} )</td>
<td>5( k )</td>
<td>25( k )</td>
<td>45( k )</td>
<td>65( k )</td>
<td>85( k )</td>
<td>105( k )</td>
<td>125( k )</td>
<td>145( k )</td>
<td>165( k )</td>
<td>185( k )</td>
<td>205( k )</td>
<td>225( k )</td>
<td>245( k )</td>
</tr>
</tbody>
</table>

(a) Which “curve” is supply and which is demand? Why?

(b) At a price of \( p = \$60 \), what will be supply and what will be demand? Will this tend to push prices higher or lower? Why?

(c) What is the equilibrium point?

15. Solve the following for \( x \)

(a) \( 7 = xe^6 \)

(b) \( 7 = 2e^{3x} \)

(c) \( \ln(x) = 7 \)

16. (Hughes-Hallett, 4e, 1.5#27) The 2004 US presidential debates questioned whether the minimum wage has kept pace with inflation. Decide the question using the following information: In 1938, the minimum wage was 25\( ^\circ \); in 2004, it was \$5.15. During that same period, inflation averaged 4.3%.

Use an exponential model to see what would have happened to 25 if it grows at 4.3% per year.

17. (Hughes-Hallett, 4e, 1.6#41) In 2000, there were about 213 million vehicles (cars and trucks) and about 281 million people in the U.S. The number of vehicles has been growing at 4% a year, while the population has been growing at 1% a year. If the growth rates remain constant, when is there, on average, one vehicle per person?

Use an exponential model for the vehicles, and an different exponential model for the people, and see when they intersect. Do this both algebraically and graphically.

18. (Hughes-Hallett, 5e, 1.7#22) An exponentially growing animal population numbers 500 at time \( t = 0 \); two years later, it is 1500. Find a formula for the size of the population \( P \) in \( t \) years and find the size of the population at \( t = 5 \).