4.6 Elasticity of Demand

Example 1.  (a) According to the USDA the price elasticity of demand for butter, at current prices and quantity sold, is 0.62. If the price of butter increase by 5% what do we expect to happen to butter sales?

(b) The average retail price of butter is about $3.50/lb right now\(^1\), and the sales are about 3.134 Million pounds per week\(^2\) If the price goes up to 3.75 what do we predict the sales to be next week?

(c) Suppose in the future the price of butter goes up dramatically; this will change the elasticity. Suppose that at a price of $5.50 we have sales of 2.023 million pounds, and then the price increases to $6 and the sales change to 1.82 million pounds. What is the new elasticity?

(d) Suppose we know the demand equation for butter: \( q = -0.406p + 4.256 \). Use the definition of elasticity to calculate \( E(5.5) \) and \( E(6) \).

\(^1\)For the week ending 11/14/2019, from the site https://www.ams.usda.gov/mnreports/dybretail.pdf
\(^2\)this is the average over the weeks 10/12/2019 – 11/9/2019, from the site https://www.ams.usda.gov/mnreports/dywdairyproductssales.pdf
Example 2. In Fall 2018, the undergraduate enrollment at Loyola University Maryland was 3886 and the tuition was $47520 per year (information taken from the 2018–2019 Loyola Catalogue). According to The Impact of Tuition Increases on Enrollment at Public Colleges and Universities by Steven W. Hemelt, Dave E. Marcotte (https://doi.org/10.3102/0162373711415261), elasticity of demand for a 4 year college is about 0.10.

(a) Will a 5% increase in tuition cause total revenue to go up or go down?
(b) Can you find a general formula for % change in $R$ as compared to $E$ and $p$, and confirm your answer?
Example 3. The 2018 demand for new cars in the US was given by

\[ q = -0.187p + 12.15 \]

where \( p \) is in thousands of dollars and \( q \) is the total number of cars sold measured in millions.\(^3\)

(a) Find the formula for revenue as a function of \( p \).
(b) Find the value of \( p \) that maximizes \( R \). What is \( R \) for this value of \( p \)? What are the units of \( R \)?

\(^3\)I based the numbers and formulas in this example upon the following facts (which I looked up): last year about 5.3 million cars were sold, at an average price of about $36,000, and at that price and quantity the price elasticity of demand for new cars is about 1.3.
Example 4. The table below shows the demand for new cars in the US in 2018, where \( p \) is in thousands of dollars and \( q \) is the total number of cars sold measured in millions.\(^4\)

<table>
<thead>
<tr>
<th>( p )</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>6.413</td>
<td>6.244</td>
<td>6.071</td>
<td>5.888</td>
<td>5.695</td>
<td>5.503</td>
<td>5.310</td>
<td>5.111</td>
<td>4.911</td>
<td>4.708</td>
</tr>
<tr>
<td>( R )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
</tbody>
</table>

(a) Fill in the row for revenue, \( R \). What are the units for \( R \)?
(b) Where is \( R \) a max?

\(^4\)I based the numbers in this table upon the following facts (which I looked up): last year about 5.3 million cars were sold, at an average price of about $36,000, and at that price and quantity the price elasticity of demand for new cars is about 1.3.
**Example 5.** The table below shows the demand for new cars in the US in 2018, where $p$ is in thousands of dollars and $q$ is the total number of cars sold measured in millions.\(^5\)

<table>
<thead>
<tr>
<th>$p$</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
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<td>6.244</td>
<td>6.071</td>
<td>5.888</td>
<td>5.695</td>
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<td>5.310</td>
<td>5.111</td>
<td>4.911</td>
<td>4.708</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fill in the row for elasticity, $E$, starting with the $p = 31$ column. To calculate $E$ in a given column, use the formula $E = \frac{\Delta q}{q} \cdot \frac{p}{\Delta p}$ where the values of $p$ and $q$ come from that column, and we calculate $\Delta q$ using the value in the current column, minus the value in the previous column, and similarly for $\Delta p$.

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\(^5\)I based the numbers in this table upon the following facts (which I looked up): last year about 5.3 million cars were sold, at an average price of about $36,000, and at that price and quantity the price elasticity of demand for new cars is about 1.3.
Example 6. The 2018 demand for new cars in the US was given by

\[ q = -0.187p + 12.15 \]

where \( p \) is in thousands of dollars and \( q \) is the total number of cars sold measured in millions.\(^6\)

(a) Using the definition of elasticity that involves derivatives, \( E = -\frac{d q}{d p} \cdot \frac{p}{q} \), find a formula for elasticity as a function of \( p \).

(b) Solve \( E = 1 \) for \( p \).

\(^6\)I based the numbers and formulas in this example upon the following facts (which I looked up): last year about 5.3 million cars were sold, at an average price of about $36,000, and at that price and quantity the price elasticity of demand for new cars is about 1.3.