4.4 Optimizing Cost and Revenue

Example 1. (Hughes-Hallett, 4e, 2.5 Ex. 3) The graph of a cost function is shown below. The graph shows total cost, with \( q \) measured in thousands. Does it cost more to make the 500th item or the 2000th? (This means just that item, not all the items 1, 2, 3, \ldots, 500, together.) At approximately what production level is marginal cost the smallest? What is the total cost at this level?
Example 2. (Hughes-Hallet, 4e, 2.5#11) Let $C(q)$ represent the cost and $R(q)$ represent the revenue, in dollars, of producing $q$ items.

(a) If $C(50) = 4300$ and $C'(50) = 24$, estimate $C(52)$.
(b) If $C'(50) = 24$ and $R'(50) = 35$, approximately how much profit is earned by the 51st item?
(c) If $C'(100) = 38$ and $R'(100) = 35$, should the company produce the 101st item? Why or why not?
Example 3. Consider the following two graphs, showing cost and revenue, and marginal cost and marginal revenue. (a) Interpret the significance of $q_1$ and $q_2$ on the graph of $R$ and $C$. (b) Identify which point is a maximum for profit, and explain.
Example 4. (Based on Hughes-Hallett, 4e, 4.4#7) The table below shows marginal cost $MC$ and marginal revenue, $MR$.

(a) Use the marginal cost and marginal revenue at a production of $q = 5000$ to determine whether production should be increased or decreased from 5000. (Explain, in writing.)

(b) Estimate the production level that maximizes profit. (Explain, in writing.)

<table>
<thead>
<tr>
<th>$q$</th>
<th>5000</th>
<th>6000</th>
<th>7000</th>
<th>8000</th>
<th>9000</th>
<th>10000</th>
<th>11000</th>
<th>12000</th>
<th>13000</th>
<th>14000</th>
<th>15000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MR$</td>
<td>60</td>
<td>58</td>
<td>56</td>
<td>55</td>
<td>54</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>$MC$</td>
<td>48</td>
<td>52</td>
<td>54</td>
<td>55</td>
<td>58</td>
<td>63</td>
<td>60</td>
<td>57</td>
<td>54</td>
<td>53</td>
<td>52</td>
</tr>
</tbody>
</table>
Example 5. (From Hughes-Hallet, 6e, 4.4#23) A farmer uses $x$ lb of fertilizer per acre at a cost of $2$ per pound, leading to a revenue of $R = 700 - 400e^{-x/100}$ dollars per acre.

(a) How many pounds of fertilizer should be applied per acre to maximize profit?
(b) What is the maximum profit on a 200 acre farm?