1.3 Average and relative rates of change

Example 1. The height of a thrown ball is given by the following function:

\[ p(t) = -4.9t^2 + 15.5t + 2 \]

where \( t \) is in seconds and \( p \) is in meters. (This is the real equation for a moving object thrown from an initial height of 2 meters, with an initial velocity of 15.5 m/s. Find the average velocity from \( t = 2.3 \) to \( t = 2.4 \).)
Example 2. In Baltimore City, Mervo High School is contained in Census Tract #090200, which is also called the Northwood Community Statistical Area (CSA). The table below shows the population in the Northwood CSA according to the Census Bureau.

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<tbody>
<tr>
<td>Population</td>
<td>3,166</td>
<td>3,577</td>
<td>3,526</td>
<td>3,736</td>
<td>3,558</td>
<td>3,302</td>
<td>2,980</td>
<td>3,027</td>
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Find rate of change in population from 2010 to 2017. Interpret your answer. What sort of impact does this rate of change have on school attendance and property values?
Example 3. According to Wikipedia, “Research conducted in the United States has indicated that, in presidential elections, American voters tend to [vote] based on the economy at large [and] on previous economic trends.” To be specific, voters are more likely to re-elect Donald Trump if the economy is getting better leading up to the election, and less likely to re-elect Donald Trump if the economy is getting worse leading up to the election. The period of time that matters most is closest to the election. One of the most important measures of the economy is jobs added/lost. Here is a graph of the number of jobs added by month for 2019:

![Graph of jobs added by month](chart.png)

where month 1 is January, month 2 is February, etc.

(a) Calculate the rate of change of jobs added: you get to pick the two different months, but you should pick months that Trump could use to argue that the economy is continuing to strengthen.

Interpret your answer, with units.

(b) Calculate the rate of change of jobs added, where you get to pick the two different months, and where you pick months that Trump’s opponent could use to argue that the economy is weakening.

Interpret your answer, with units.
Example 4. Let $f(x)$ be defined by the graph below

For each of these $x$-values, does $f(x)$ appear to be increasing, decreasing, or neither?

$$x = -1, 0, 1, 1.5, 2, 2.5, 3$$
Example 5. Let \( f(x) \) be defined by the graph below.

Over which intervals does it appear that \( f(x) \) is increasing? Decreasing? In the part of the graph that we can see, where are the maximum and minimums of the graph?
Example 6. (a) In Fall 2017, the undergraduate enrollment at Loyola University Maryland was 3886 and in Fall 2018 the undergraduate enrollment at Loyola was 4271. What was the relative change in enrollment? Give your answer in a complete sentence.

(b) In Fall 2018, the undergraduate full time tuition at Loyola University was $46160 and in Fall 2018 the tuition was $47520. What was the relative change in tuition? Give your answer in a complete sentence.
Extra examples

Example 7. (Hughes-Hallett, 4e, 1.3#24) The table below shows the production of tobacco in the US, in millions of pounds.

(a) What is the average rate of change in tobacco production between 1996 and 2003? Give units and interpret your answer in terms of tobacco production.

(b) During this seven-year period, is there any interval during which the average rate of change was positive? If so, when?

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</thead>
<tbody>
<tr>
<td>Production</td>
<td>1517</td>
<td>1787</td>
<td>1480</td>
<td>1293</td>
<td>1053</td>
<td>991</td>
<td>879</td>
<td>831</td>
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Example 8. (Hughes-Hallett, 3e, 1.3#26) The table below shows the sales, $S$, in millions of dollars, of Intel Corporation, a leading manufacturer of integrated circuits:

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
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<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>26,273</td>
<td>29,389</td>
<td>33,726</td>
<td>26,539</td>
<td>26,764</td>
<td>30,141</td>
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</tbody>
</table>

(a) What is the average rate of change from 1998 to 2003? Interpret its units and meaning.

(b) Assuming that the change continues at the same rate as in part (a), when will sales reach 40,000 million dollars?

(c) Over which intervals does it appear that the function $S$ is increasing?