Midterm 2 preview, MA 151, Applied Calculus, Spring 2014

Your name: _____

Your student number: _____

Showing work. Each answer must include step-by-step work on your page (except for # 5). Even calculations which will be done in a calculator, should first appear as formulas on the page (here, "formulas" can mean function expressions, like $C(50) - C(49)$ or $f(100) + \Delta x$ or $-4(5^3) - e^5$).

Time management. Probably not everyone will have enough time to do every problem correctly. I think it is better, and that you will get a better score, if you (1) skip the hardest problems until later, (2) work carefully and write more complete steps so that you don't make mistakes and you get better partial credit, and you can see which part of your work is correct, etc.

Problem	Possible points	Points received
1	17	
2	17	
3	17	
4	17	
5	17	
6	17	
Total	102	

Please sign the following pledge:

On my honor I have neither given nor received any aid on this exam; I have upheld the ideals of the honor code.

Signature _____

1. (Hughes-Hallett 4.2 #22) An exponentially growing animal population numbers 500 at time $t = 0$; two years later, it is 1500. Find a formula for the size of the population in t years and find the size of the population at $t = 5$.

2. Let $f(x) = \sqrt{x} + x$ and $g(x) = e^x + x$.

   (a) Find $f(x)g(x)$

   (b) Find $f(x)/g(x)$.

   (c) Find $f(g(x))$.

   (d) Find $g(f(x))$.

3. Find a function $f(x)$ that is proportional to the 3rd power of x and that satisfies $f(2) = 10$.

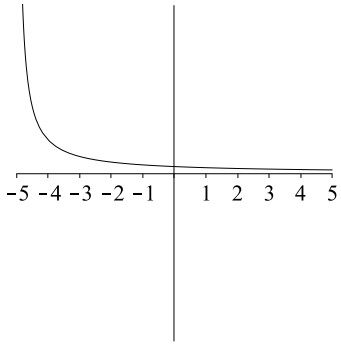
4. Let $f(x) = xe^x$. Find an approximation of $f'(2.5)$ by filling in the following table using the same rules as on the quiz[1].

x	$f(x)$	$\dfrac{f(2.5) - f(x)}{2.5 - x}$

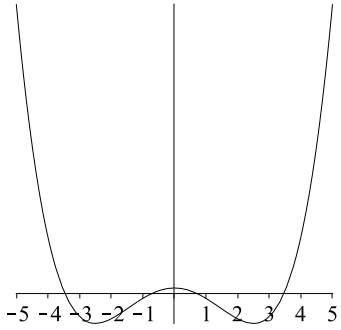
[1] You should plug numbers in for x that are close to 2.5. They should be close enough that you get two approximations of $f'(2.5)$ that are the same to the first decimal place. You should always use at least 8 digits in every step of the calculation.

5. Shown below are eight functions. Four of them are derivatives of the other four. Identify which function is the derivative of which other function; justify each answer by referring to a specific x-value, indicate whether the function is flat/increasing/decreasing and whether the derivative is 0/positive/negative at this x-value. For instance, you could say "(u) is the derivative of (v), because at $x = 1$ we have (v) is increasing and (u) is positive," or you could be more abbreviated and say the same thing with "u=v$'$, @ $x = 1$, v$\uparrow$, u=+."

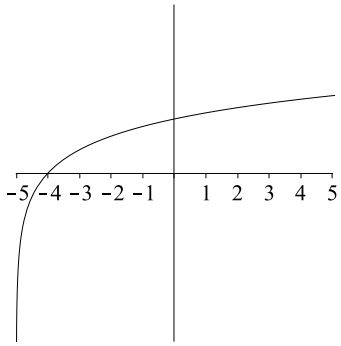
(a)



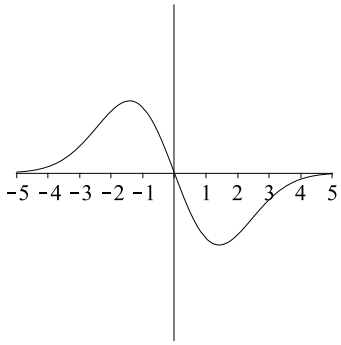
(b)



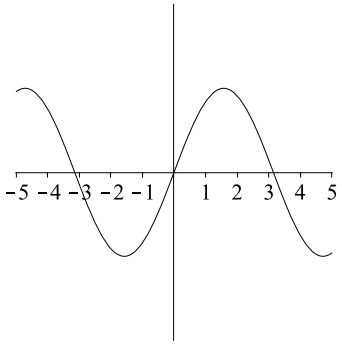
(c)



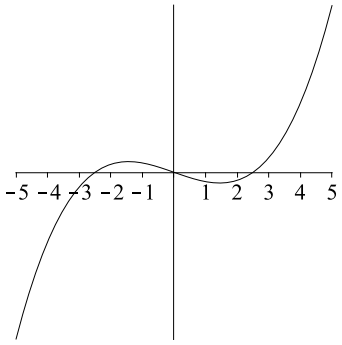
(d)



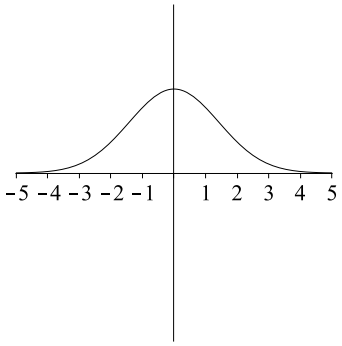
(e)



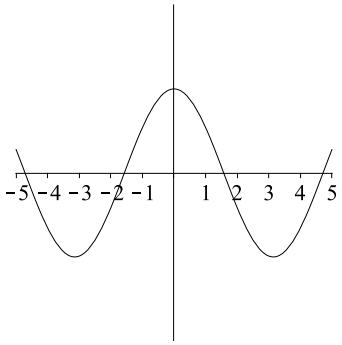
(f)



(g)



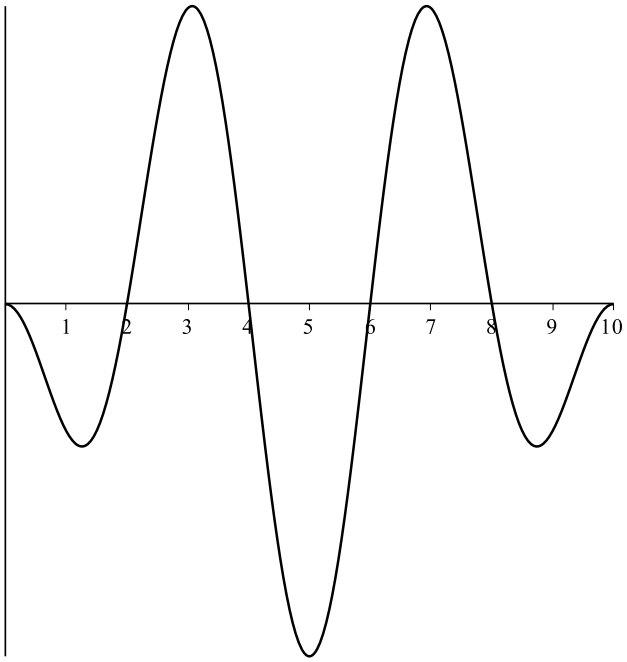
(h)



6. Leibniz notation and interpretation. (Based on Hughes-Hallett, 4e, #10) On a certain day, CBS Evening News had a 4.3 rating. (Ratings measure the number of viewers.) News executives estimated that on that day a 0.1 drop in the ratings for the CBS Evening News corresponds to a $\$5.5$ million drop in revenue. Let R be the revenue and p be the ratings.

   Express the information in this problem as a derivative using Leibniz notation. Specify the units, the point at which the derivative is evaluated, and interpret your answer.

7. Let $f(x)$ be defined by the graph below



   For each quantity below, indicate whether it is $+$, $-$ or 0

$f(0.75) =$	$f'(0.75) =$	$f''(0.75) =$
$f(1) =$	$f'(1) =$	$f''(1) =$
$f(1.25) =$	$f'(1.25) =$	$f''(1.25) =$
$f(2.25) =$	$f'(2.25) =$	$f''(2.25) =$
$f(3) =$	$f'(3) =$	$f''(3) =$
$f(4) =$	$f'(4) =$	$f''(4) =$
$f(5) =$	$f'(5) =$	$f''(5) =$
$f(6) =$	$f'(6) =$	$f''(6) =$
$f(7.75) =$	$f'(7.75) =$	$f''(7.75) =$
$f(8.75) =$	$f'(8.75) =$	$f''(8.75) =$
$f(9.25) =$	$f'(9.25) =$	$f''(9.25) =$

8. Suppose $f(23) = 175$ and $f'(23) = 1.5$.

   (a) Find an estimate for $f(26)$.

   (b) Suppose now that you know, for the same function f, that $f''(x)$ is positive for all x. Is your estimate in (a) too high or too low? Justify your answer.

9. Shown below is the graph of a total cost function of making q items. Use the graph to answer the following questions:

   (a) Find the total cost of making 50 items.

   (b) Estimate the marginal cost when $q = 50$; give units.