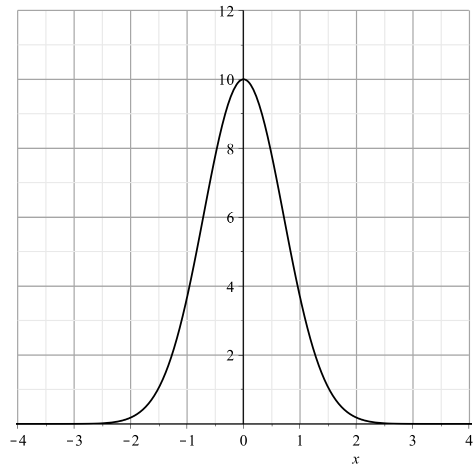
MA 151, Spring 2014, Midterm 1 preview solutions:

1. This problem refers to the graph below.

   (a) Find $f(1)$, $f(-0.5)$.

   (b) Solve $f(x) = 2$.



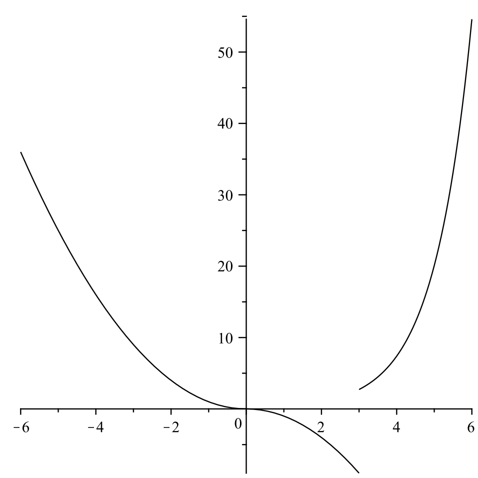
Solution. (a) $f(1) \approx 3.75$ (start by finding $x = 1$ on the graph, and then see what the y-value is there) and $f(-0.5) \approx 7$.

(b) $x \approx -1.25$ and $x \approx 1.25$ (start by finding $y = 1$ on the graph, this happens in two places, and then see what the x-values are).

2. This problem refers to the graph below.

   (a) On what intervals is the graph increasing? On what intervals is it decreasing?

   (b) On what intervals is it concave up? On what intervals is it concave down?



Solution. (a) $f(x)$ is increasing from 3 to 6. It is decreasing from -6 to 3.

(b) $f(x)$ is concave up from -6 to 0, and from 3 to 6. It is concave down from 0 to 3.

3. Solve $f(x) = 3$ where $f(x) = 12x - 7$.

Solution. $\boxed{\text{DO NOT DO THIS } f(3) = 12(3) - 7}$. Pay close attention to the input and output in function notation:

$$\underset{\text{input}}{\underset{\uparrow}{f(x)}} = \underset{\text{output}}{\underset{\uparrow}{3}} \qquad \text{and} \qquad \underset{\text{input}}{\underset{\uparrow}{f(x)}} = \underbrace{12x - 7}_{\text{output}}$$

So, hopefully you know what two things to set equal to each other:

$$\begin{aligned} 3 &= 12x - 7 \\ 10 &= 12x \\ x &= \frac{10}{12} \\ &= 5/6 \end{aligned}$$

4. Find the equation of the line through the points $(3, 7)$ and $(-1, -10)$.

$$\begin{aligned} y &= m(x - x_0) + y_0 \\ m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{7 - (-10)}{3 - (-1)} \\ &= 17/4 \\ y &= \frac{17}{4}(x - 3) + 7 \end{aligned}$$

5. (Hughes-Hallett, 4e, 1.2#13) A company rents cars at \$40 a day and 15 cents a mile. It's competitor's cars are \$50 a day and 10 cents a mile.

   (a) For each company, give a formula for the ecost of renting a car for a day as a function of the distance traveled.

   (b) On the same axes, graph both functions.

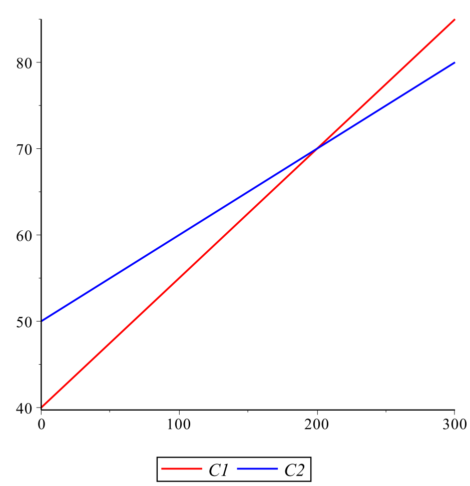
   (c) How should you decide which company is cheaaper?

Solution. (a) Let $d =$ distance travelled.

Company 1: $C1 = 40 + 0.15d$.

Company 2: $C2 = 50 + 0.10d$.

(b)



(c) Which company is cheaper depends on how many miles you will drive. For $d < 200$, company 1 is cheaper. For $d > 200$, company 2 is cheaper.

6. (Hughes-Hallett, 4e, 1.3# 14) When a deposit of $1000 is made into an account poying 8% interest, compounded annually, the balance, $$B$, in the account after t years is given by $B = 1000(1.08)^t$. Find the average rate of change in the balance over the interval $t = 0$ to $t = 5$. Give units and interpret your answer in terms of the balance in the account.

   Solution.

$$\begin{aligned} \text{av.rt.ch} &= \frac{B(5) - B(0)}{5 - 0} \\ &= \frac{1000(1.08)^5 - 1000(1.08)^0}{5} \\ &= \frac{467.33}{5} \\ &= 98.87 \end{aligned}$$

   On average, the bank account balance is growing by $98.87 each year.

7. Suppose a falling rock has position given by the following formula:

$$p(t) = -4.9t^2 + 13t + 10,$$

   where p is measured in meters and t in seconds. Find the average velocity from $t = 1$ to $t = 2$ of the rock, including units.

   Solution. The average velocity is the average rate of change of position. It equals $\frac{\Delta\text{position}}{\Delta\text{time}}$.

$$\begin{aligned} \frac{p(2) - p(1)}{2 - 1} &= \frac{-4.9(2^2) + 13(2) + 10 - (-4.9(1^2) + 13(1) + 10)}{1} \\ &= -1.7m/s \end{aligned}$$

8. A company is going to make a new kind of glue. To set up the factor, pay for the building, buy the machines, etc. will cost $1,225,000. Each tube of glue will cost $0.50 to make. They will sell each tube for $2.

   (a) Find a formula for the cost function, the revenue function, and the profit function.

   (b) Find the break even point.

   Solution. (a)

$$\begin{aligned} C(q) &= 1225000 + 0.5q \\ R(q) &= 2q \\ \pi(q) &= R(q) - C(q) \\ &= 2q - (1225000 + 0.5q) \\ &= 1.5q - 1225000 \end{aligned}$$

   (b)

$$\begin{aligned} \pi(q) &= 0 \\ 1.5q - 1225000 &= 0 \\ 1.5q &= 1225000 \\ q &= \frac{1225000}{1.5} \\ &\approx 816666 \end{aligned}$$

9. A company is making all electric sports cars. Their cost function is $C(q) = 10 + 1.5q$ where q is the number of cars they make and C is measure in millions of dollars.

   (a) Suppose the company can sell 1 car if the price it at $p = 0.5$ (i.e. half a million dollars), and they can sell 10 cars if they price it at $p = 0.1$ (i.e. $100,000). Assume that demand is linear and write a formula for the demand function.

   (b) Combine your answer to part (a) with the cost function to have a formula for $C(p)$, i.e. cost as a function of price.

   Solution. (a) The demand function gives q, the quantity sold, as a function of p, the price. Thus we want $q =$ stuff with p. In other words, we want to fill in the blanks: $q = \_\_p + \_\_$. The data points are $(0.5, 1)$, $(0.1, 10)$.

$$\begin{aligned} y &= m(x - x_0) + y_0 \\ m &= \frac{10 - 1}{0.1 - 0.5} \\ &= \frac{9}{-0.4} = -22.5 \\ q &= -22.5(p - 0.1) + 10 \end{aligned}$$

   (b) We want to eliminate q from $C(q) = 10 + 1.5q$. From (a), we know that q equals something involving p, so we just replace q with the formula we found

$$C(p) = 10 + 1.5(\underbrace{-22.5(p - 0.1) + 10}_{q}) = 23.875 - 33.75p$$

10. (Hughes-Hallett, 4e, 1.4#28) The demand curve for a product is given by $q = 120,000 - 500p$ and the supply curve is given by $q = 1000p$ for $0 \le q \le 120,000$, where price is in dollars.

   (a) At a price of $100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?

   (b) Find the equilibrium price and quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to the equilibrium?

   Solution. (a) Consumers: $120,000 - 500(100) = 70,000$.

   Suppliers: $1000(100) = 100,000$.

   The market will tend to push prices down. There is an oversupply and the only way to get rid of the extra things is to lower the price.

   (b) Equilibrium is where the supply and demand intersect:

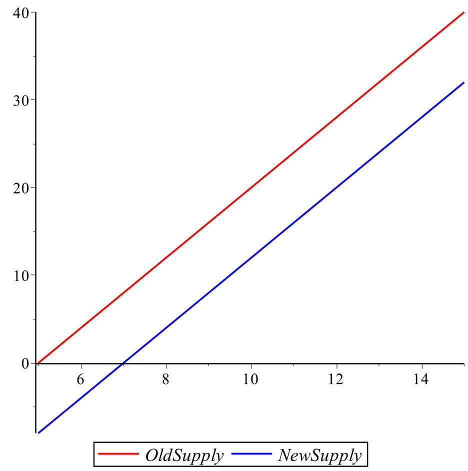
$$\begin{aligned} 120000 - 500p &= 1000p \\ 120000 &= 1500p \\ p &= 120000/1500 \\ &= 80 \end{aligned}$$

   The price before was 100, and it will tend to move towards 80, which confirms what we said in part (a).

11. (Hughes-Hallet, 4e, 1.4#35) A supply curve has equation $q = 4p - 20$, where p is price in dollars. A $2 tax is imposed on suppliers. Find the equation of the new supply curve. Sketch both curves.

Solution. Old supply: $q = 4p - 20$. But now p changes for the suppliers. They don't keep all of the price p, but rather they keep only $p - 2$. We substitute $p - 2$ into the equation:

$$q = 4(p - 2) - 20 = 4p - 28.$$



12. Solve the following for x

    (a) $7 = xe^6$

    (b) $7 = 2e^{3x}$

    (c) $\ln(x) = 7$

Solution. (a) $x = \dfrac{7}{e^6}$

(b)

$$\begin{aligned} \frac{7}{2} &= e^{3x} \\ \ln\left(\frac{7}{2}\right) &= \ln\left(e^{3x}\right) \\ \ln(7/2) &= 3x \\ x &= \frac{\ln(7/2)}{3} \end{aligned}$$

(c)

$$\begin{aligned} \ln(x) &= 7 \\ e^{\ln(x)} &= e^7 \\ x &= e^7 \end{aligned}$$

13. (Hughes-Hallet, 4e, 1.5#27) The 2004 US presidential debates questioned whether the minimum wage has kept pace with inflation. Decide the question using the following information: In 1938, the minimum wage was 25¢; in 2004, it was \$5.15. During that same period, inflation averaged 4.3%.

Solution. There are a few different, correct, ways to approach this. I think the simplest to understand is simply to compare \$5.15 with what would have happened to 25 if the 25 had grown at 4.3% per year.

$$\begin{aligned}\text{Inflation adjusted value} &= 0.25(1.043)^t, \quad t = \text{years since 1938}\\ &= 0.25(1.043)^{66}\\ &= 4.02.\end{aligned}$$

So, if the original 25¢ had grown at 4.3% per year, it would be \$4.02 in 2004. Since the actual minimum wage was \$5.15, the actual minimum wage had more than kept pace with inflation.

14. (Hughes-Hallet, 4e, 1.6#41) In 2000, there were about 213 million vehicles (cars and trucks) and about 281 million people in the U.S. The number of vehicles has been growing at 4% a year, while the population has been growing at 1% a year. If the growth rates remain constant, when is there, on average, one vehicle per person?

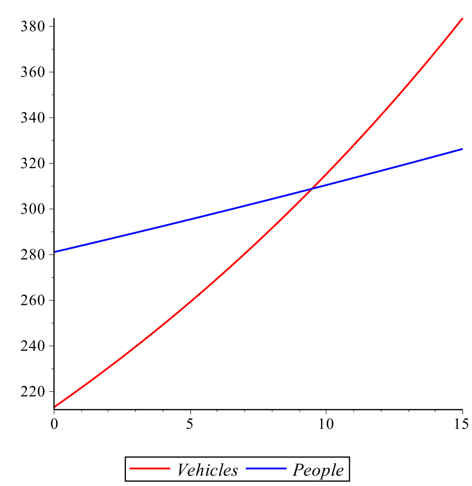
Solution. As in the previous problem, there are a few different, correct, ways to approach this. Again, I'll pick what I hope is the simplest approach to understand.

$$\begin{aligned}\text{number of vehicles} &= 213(1.04)^t\\ \text{number of people} &= 281(1.01)^t\end{aligned}$$

(where both are measured in millions, and t is the number of years since 2000).

Our goal is to find when the number of vehicles equals the number of people. We'll do this graphically and algebraically.

Graphically, we intersect the two curves



Algebraically, we solve an equation

$$\begin{aligned}
213(1.04)^t &= 281(1.01)^t \\
\frac{(1.04)^t}{(1.01)^t} &= \frac{281}{213} \\
\left(\frac{1.04}{1.01}\right)^t &= 1.319 \\
(1.0297)^t &= 1.319 \\
\ln\left((1.0297)^t\right) &= \ln(1.319) \\
t\ln(1.0297) &= \ln(1.319) \\
t &= \frac{\ln(1.319)}{\ln(1.0297)} \\
&\approx 9.46
\end{aligned}$$