Example 1. (Based on Hughes-Hallett, 4e, 3.3 Example 1) The amount of gas, $G$, in gallons, consumed by a car depends on the distance traveled, $s$, in miles. But, suppose we want to know how much gas is consumed each hour, not each mile? Well, the distance traveled, $s$ depends on the time traveled, $t$, in hours. Let 0.05 gallons of gas be consumed for each mile traveled, and suppose that the car is traveling at 30 mi/hr. How fast is gas being consumed? Give units.

Solution. The information we are given can be summarized this way:

\[ \frac{dG}{ds} = 0.05 \text{ gal/mi}, \quad \frac{ds}{dt} = 30 \text{ mi/hr}, \]

and we want to find this:

\[ \frac{dG}{dt} = ? \]

If we give these numbers and units the usual interpretation, we have this:

- We use 0.05 gallons by driving one mile,
- We drive 30 miles in one hour,
- How many gallons will we use in one hour?

The right way to answer this question is to combine the given information with multiplication:

\[
\text{gallons in one hour} = 30 \text{ miles in one hour} \times 0.05 \text{ gallons in one mile} = 1.5 \text{ gal/hr}.
\]

We can summarize this calculation in Leibniz notation this way:

\[
\frac{dG}{dt} = \frac{dG}{ds} \cdot \frac{ds}{dt}.
\]
Example 2. Write each of the following functions as a function of $z$, where $z$ is the “inside” function.

(a) $y = \sqrt{x^2 + 2x}$
(b) $y = 5(2x + 7)^8$
(c) $y = \frac{11}{x^2 + 1}$
(d) $y = -7.2e^{x^2}$
(e) $y = \frac{1}{2}\ln(3x^2 + 5)$
Example 3. Find the derivatives of the following functions. Use $z$ for the inside function, and use the Leibniz notation for the chain rule.

(a) $y = 5(-3x^2 + 2x + 7)^{11}$.
(b) $y = \frac{7}{3} \ln(x^3 + x)$. 
Example 4. Find the following derivatives

(a) \( \frac{d}{dx} (3x - 7)^{11} \)
(b) \( \frac{d}{dx} \sqrt{1.9x - 4.3} \)
(c) \( \frac{d}{dx} \frac{1}{10x - 4} \)
(d) \( \frac{d}{dx} e^{1.03x - 1} \)
(e) \( \frac{d}{dx} \ln(2x + e^2) \)