Chapter 4

Using the Derivative

4.1 Local Max and Mins

Definition. Let $x = c$ be in the domain of $f(x)$.

$x = c$ is a	local maximum	if $f(x) \leq f(c)$ for all x near c. (we allow endpoints)
$x = c$ is a	local minimum	if $f(x) \geq f(c)$ for all x near c. (we allow endpoints)

Definition. If $x = c$ is in the domain of f and $f'(c) = 0$, then we call $x = c$ a **critical point**. We also call the (x, y)-point $(c, f(c))$ a critical point. We call $f(c)$ the **critical value**.

Theorem 1 (First derivative test)**.** To find the local max/mins of a function $f(x)$ do the following.

1. First find the critical points.
2. Figure out whether $f'(x)$ is $+$ or $-$ on each side of each critical point (four cases, lots of pictures):

$f'(x) = +, -$ left of c	right of c	outcome
$+$	$-$	$\Rightarrow x = c$ local max
$-$	$+$	$\Rightarrow x = c$ local min
$+$	$+$	$\Rightarrow x = c$ neither
$-$	$-$	$\Rightarrow x = c$ neither

Theorem 2 (Second derivative test)**.** To find the local max/mins of a function $f(x)$ try the following.

1. First find the critical points.
2. Figure out whether $f''(c)$ is $+$ or $-$ (three cases):

$f''(c)$	outcome
$+$	local min
$-$	local max
0 or DNE	test says nothing