5.3 The Definite Integral as Area

Fact.

$$\int_a^b f(x)\,dx = \boxed{\begin{array}{l}\text{Area under } f(x)\\ \text{from } x=a \text{ to } x=b\end{array}} \qquad \text{(when } f(x) > 0 \text{ and } a < b\text{)}$$

Fact. If $f(x)$ is sometimes positive, and sometimes negative, then

$$\int_a^b f(x)\,dx = \boxed{\begin{array}{l}\text{Area above the } x\text{-axis}\\ \text{minus area below } x\text{-axis}\\ \text{(between graph of } f(x) \text{ and the } x\text{-axis}\\ \text{ and from } a \text{ to } b\text{)}\end{array}} \qquad (a < b)$$

5.4 The Interpretations of the Definite Integral

Fact. Suppose ☺ is some quantity of something. Suppose $f(t)$ is the rate of change of ☺. Then

- units of $f(t) = \dfrac{\text{units of ☺}}{\text{units of time}}$
- Δ☺ $\approx f(t) \times \Delta t$
- $\boxed{\begin{array}{l}\text{Total change of ☺}\\ \text{from } t=a \text{ to } t=b\end{array}} = \int_a^b f(t)\,dt$
- "Total change" means "net change". In other words, it's the increase in ☺ minus the decrease in ☺.

5.5 The Fundamental Theorem of Calculus

Theorem 1 (The Fundamental Theorem of Calculus)**.**

$$F(b) - F(a) = \int_a^b F'(t)\,dt$$

Fact. If $C(q)$ is a cost function then

$$\boxed{\begin{array}{l}\textbf{Total change in cost}\\ \text{from } q=a \text{ to } q=b\end{array}} = C(b) - C(a) = \int_a^b C'(q)\,dq$$

One special case of the preceding formula is when $a = 0$:

$$C(b) = C(0) + \int_0^b C'(q)\,dq \qquad (C(0) = \textbf{fixed cost})$$

Note that $C(b)$ is the total cost of making b units. This is different than $\int_0^b C'(q)\,dq$ which is called the **total variable cost** of making b units.