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Chapter 0

Review: Lines, Fractions, Exponents

0.1 Lines

0.2 Fractions

0.3 Rules of exponents

Example 1. Find the equation of the line through the points $(1, 2)$ and $(3, 4)$.

Chapter 1

Functions and Change

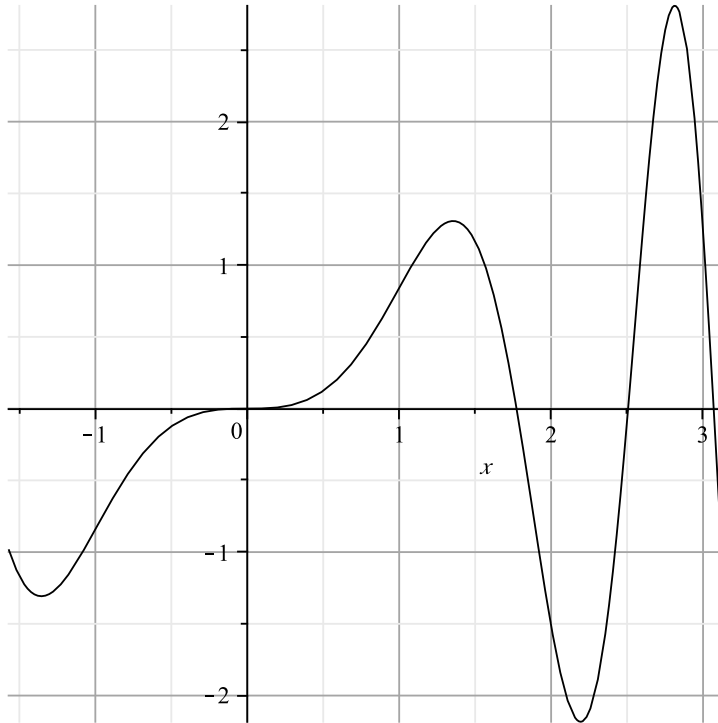
1.1 Functions

Example 1. Let $f(x)$ be defined by the table of numbers below.

x	1	2	2.5	2.9	3.1	3.5	4
$f(x)$	2	3	3.1	4	1	2.9	3

- (a) Find $f(1)$. Find $f(4)$.
- (b) Is it possible to find $f(1.5)$?
- (c) Solve $f(x) = 1$. Solve $f(x) = 3$.

Example 2. Let $f(x)$ be the function defined by the graph below



- (a) Find $f(0)$, $f(1)$, $f(1.5)$, $f(2)$.
(b) Solve $f(x) = 0$; solve $f(x) = 1$ (find all solutions)

Example 3. Let $f(x) = 3x - 7$.

- (a) Find $f(1)$. Find $f(2)$.
- (b) Solve $f(x) = 1$; solve $f(x) = 2$ (find all solutions).

Example 4. Let $f(x) = -2x + 5$.

- (a) Find $f(3)$.
- (b) Solve $f(x) = 3$.

1.2 Linear Functions

Example 1. (Hughes-Hallett, 3e, 1.2#24) The number of species of coastal dune plants in Australia decreases as the latitude, in $^{\circ}\text{S}$, increases. There are 34 species at 11°S and 26 species at 44°S .

- (a) Find a formula for the number, N , of species of coastal dune plants in Australia as a linear function of the latitude, ℓ in $^{\circ}\text{S}$.
- (b) Give units for and interpret the slope and the vertical intercept of this function.
- (c) Graph this function between $\ell = 11^{\circ}\text{S}$ and $\ell = 44^{\circ}\text{S}$. (Australia lies entirely within these latitudes.)

Example 2. (Hughes-Hallett, 3e, 1.2#25) A controversial 1992 Danish study reported that men's average sperm count has decreased from 113 million per milliliter in 1940 to 66 million per milliliter in 1990.

- (a) Express the average sperm count, S , as a linear function of the number of years, t , since 1940.
- (b) A man's fertility is affected if his sperm count drops below about 20 million per milliliter. If the linear model found in part (a) is accurate, in what year will the average male sperm count fall below this level?

1.3 Rates of change

Example 1. (Hughes-Hallett, 3e, 1.3#11) The table below shows the production of tobacco in the US, in millions of pounds.

- (a) What is the average rate of change in tobacco production between 1996 and 2003? Give units and interpret your answer in terms of tobacco production.
- (b) During this seven-year period, is there any interval during which the average rate of change was positive? If so, when?

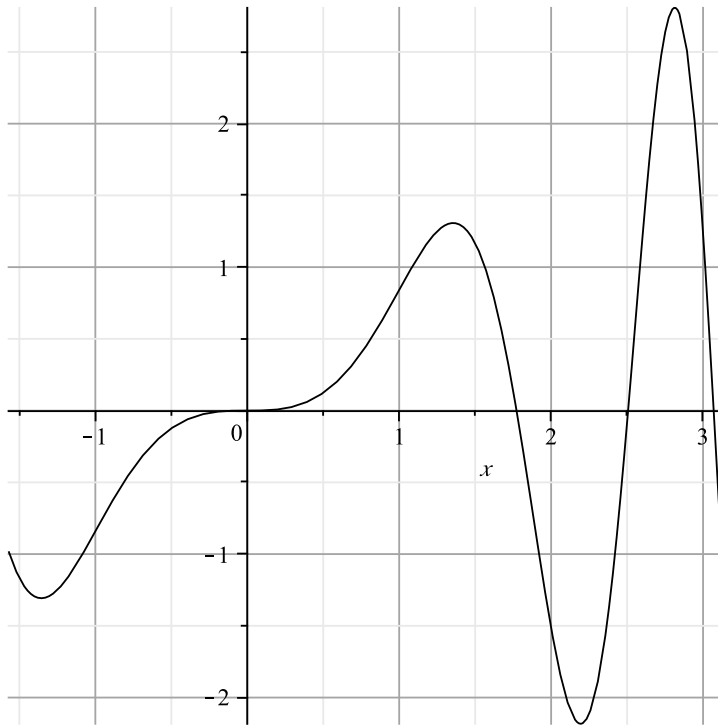
Year	1996	1997	1998	1999	2000	2001	2002	2003
Production	1517	1787	1480	1293	1053	991	879	831

Example 2. (Hughes-Hallett, 3e, 1.3#26) The table below shows the sales, S , in millions of dollars, of Intel Corporation, a leading manufacturer of integrated circuits:

Year	1998	1999	2000	2001	2002	2003
S	26,273	29,389	33,726	26,539	26,764	30,141

- What is the average rate of change from 1998 to 2003? Interpret its units and meaning.
- Assuming that the change continues at the same rate as in part (a), when will sales reach 40,000 million dollars?
- Over which intervals does it appear that the function S is increasing?

Example 3. Let $f(x)$ be defined by the graph below



- (a) Over which intervals does it appear that $f(x)$ is increasing? Decreasing?
(b) Over which intervals does it appear that $f(x)$ is concave up? Concave down?

1.4 Applications of Functions to Economics