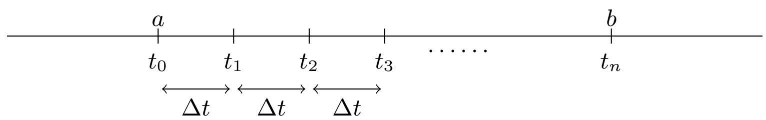
5.2 The Definite Integral

Definition. If $v(t)$ is velocity, and positive, then

$$\boxed{\begin{array}{l}\text{distance}\\ \text{travelled from}\\ t=a\text{ to }t=b\end{array}} \approx v(t_0)\Delta t + v(t_1)\Delta t + \cdots + v(t_{n-1})\Delta t, \tag{5.1}$$

$$\boxed{\begin{array}{l}\text{distance}\\ \text{travelled from}\\ t=a\text{ to }t=b\end{array}} \approx v(t_1)\Delta t + v(t_2)\Delta t + \cdots + v(t_n)\Delta t. \tag{5.2}$$

We call the formula in Equation 5.1 a **Left Hand Riemann Sum** and the formula in Equation 5.2 a **Right Hand Riemann Sum**. The interval $[a, b]$ is broken up into pieces of width Δt, and t_0, t_1, $\ldots$, t_n are the endpoints of these pieces, and n is the number of pieces:



We define the *exact* distance to be the number obtained (as a limit) by using smaller and smaller Δt, which means more and more pieces:

$$\boxed{\begin{array}{l}\text{distance}\\ \text{travelled from}\\ t=a\text{ to }t=b\end{array}} = \text{sum of an infinite number of terms like } v(t_i) \times \Delta t \text{ in a Riemann Sum.}$$

Finally, we invent a symbol that stands for the number you get by adding more and more terms as just described:

$$\int_a^b v(t)\,dt = \text{sum of infinite number of terms like } v(t_i) \times \Delta t \text{ in a Riemann Sum.}$$

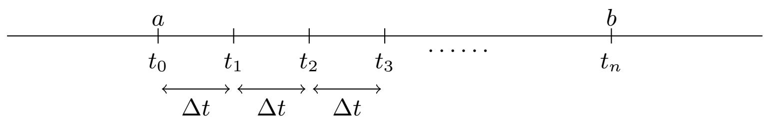
We call $\int_a^b v(t)\,dt$ the **definite integral of** $v(t)$ **from** a **to** b.

Definition. If $f(t)$ is any function we define

$$\int_a^b f(t)\,dt \approx f(t_0)\Delta t + f(t_1)\Delta t + \cdots + f(t_{n-1})\Delta t, \tag{5.3}$$

$$\int_a^b f(t)\,dt \approx f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_n)\Delta t. \tag{5.4}$$

We call the formula in Equation 5.3 a **Left Hand Riemann Sum** and the formula in Equation 5.4 a **Right Hand Riemann Sum**. The interval $[a, b]$ is broken up into pieces of width Δt, and t_0, t_1, …, t_n are the endpoints of these pieces, and n is the number of pieces:



The approximation in the previous equations is made exact (as a limit) by using smaller and smaller Δt, which means more and more pieces:

$$\int_a^b f(t)\,dt = \text{sum of an infinite number of terms like } f(t_i) \times \Delta t \text{ in a Riemann Sum.}$$

We call $\int_a^b f(t)\,dt$ the **definite integral of** $f(t)$ **from** a **to** b.