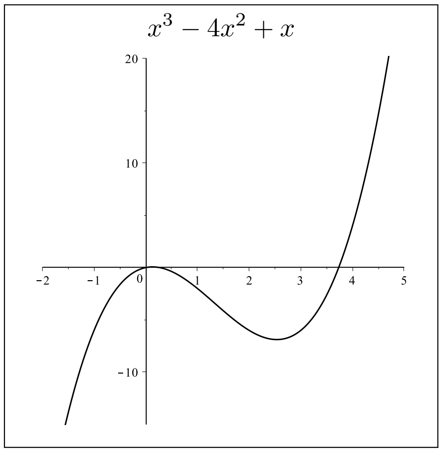
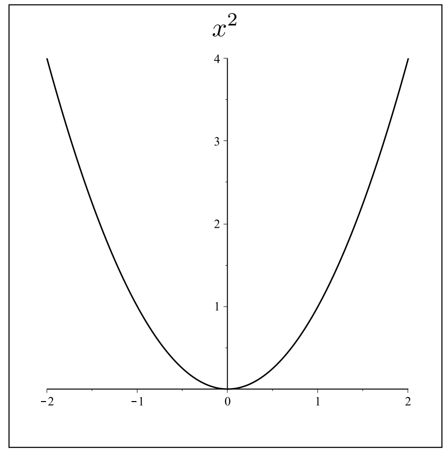
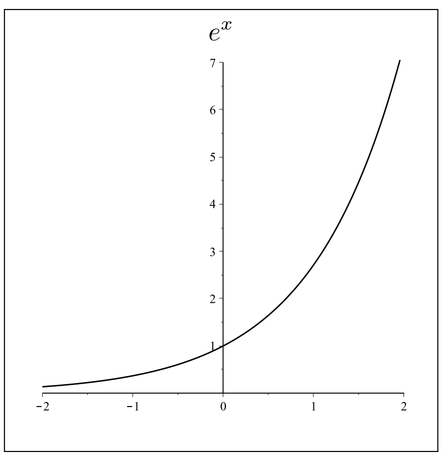
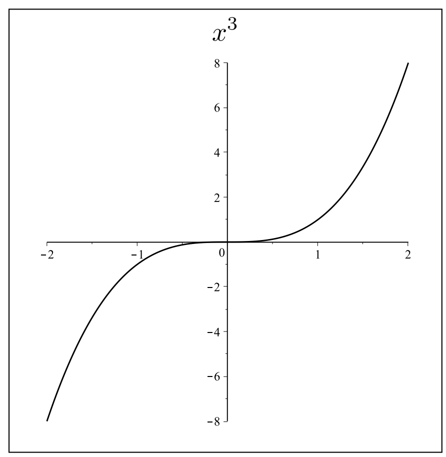
Basic Functions

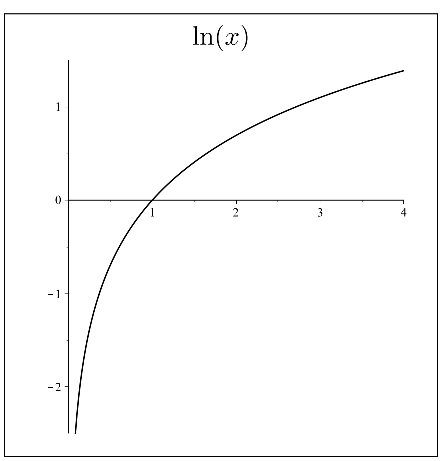
1. Identify each of the graphs below as one of the following functions:

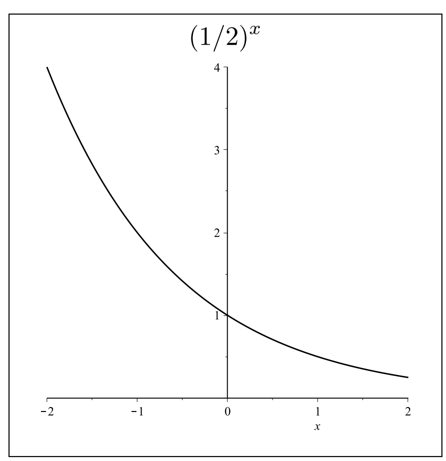


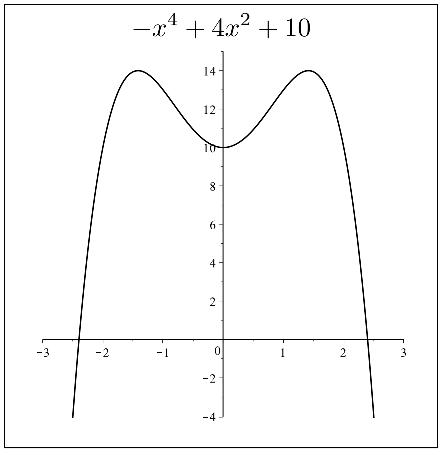


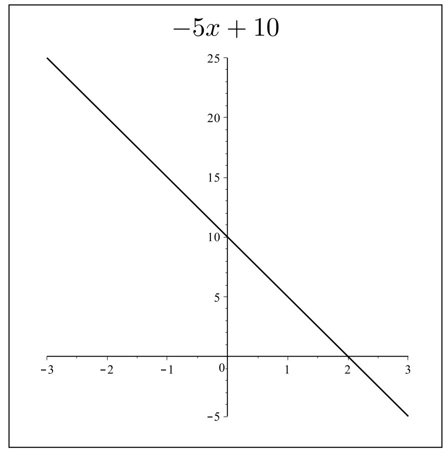


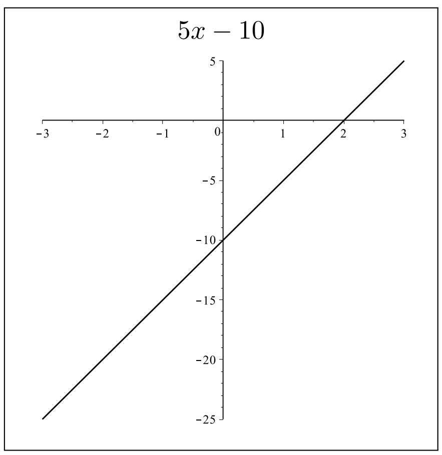












Function notation, solving function equations

2. (This problem as a whole is obviously too long for the final: but I could have a problem with a couple of parts like this.)

   (a) Let $f(x) = 3x + 5$. Solve $f(x) = 3$.

   Solution:

$$\begin{aligned} 3x + 5 &= 3 \\ 3x &= -2 \\ x &= -2/3 \end{aligned}$$

   (b) Let $f(x) = x^2$. Solve $f(x) = 5$.

   Solution:

$$\begin{aligned} x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$

(c) Let $f(x) = 2x^3 - 5$. Solve $f(x) = -10$.

Solution:

$$\begin{aligned} 2x^3 - 5 &= -10 \\ 2x^3 &= -5 \\ x^3 &= -\frac{5}{3} \\ x &= \sqrt[3]{-\frac{5}{3}} \\ &= -\sqrt[3]{5/3} \end{aligned}$$

(d) Let $f(x) = x^2 - 2x - 15$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned} x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \\ (x-5) = 0 \text{ or } (x+3) &= 0 \\ x = 5 \text{ or } &-3 \end{aligned}$$

(e) Let $f(x) = x^2 - 2x - 14$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned} x^2 - 2x - 14 &= 0 \\ x &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-14)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4+56}}{2} \\ &= \frac{2 \pm \sqrt{60}}{2} \end{aligned}$$

(f) Let $f(x) = e^x - 2x^2e^x$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned} e^x - 2x^2e^x &= 0 \\ e^x(1-2x^2) &= 0 \\ e^x = 0 \text{ or } (1-2x^2) &= 0 \\ e^x = 0 \text{ is impossible} & \\ 1 - 2x^2 &= 0 \\ 2x^2 &= 1 \\ x^2 &= 1/2 \\ x &= \pm\sqrt{1/2} \end{aligned}$$

(g) Let $f(x) = \frac{1}{x} + 1$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned}\frac{1}{x} + 1 &= 0\\ \frac{1}{x} &= -1\\ \frac{1}{x} &= \frac{-1}{1}\\ 1 &= -x \text{ (cross multiply)}\\ x &= -1\end{aligned}$$

(h) Let $f(x) = \frac{2}{x^2} - 5$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned}\frac{2}{x^2} - 5 &= 0\\ \frac{2}{x^2} &= 5\\ \frac{2}{x^2} &= \frac{5}{1}\\ 2 &= 5x^2 \text{ (cross multiply)}\\ x^2 &= 2/5\\ x &= \pm\sqrt{2/5}\end{aligned}$$

(i) Let $f(x) = 3e^x - 5$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned}3e^x - 5 &= 0\\ 3e^x &= 5\\ e^x &= 5/3\\ \ln(e^x) &= \ln(5/3)\\ x &= \ln(5/3)\end{aligned}$$

(j) Let $f(x) = 4e^{2x} - 3$. Solve $f(x) = 0$.

Solution:

$$\begin{aligned}4e^{2x} - 3 &= 0\\ 4e^{2x} &= 3\\ e^{2x} &= 3/4\\ \ln(e^{2x}) &= \ln(3/4)\end{aligned}$$

$$2x = \ln(3/4)$$
$$x = \frac{1}{2}\ln(3/4)$$

(k) Let $f(x) = \ln(x) + 5$. Solve $f(x) = 0$.

Solution:

$$\ln(x) + 5 = 0$$
$$\ln(x) = -5$$
$$e^{\ln(x)} = e^{-5}$$
$$x = e^{-5}$$

(l) Let $f(x) = 2\ln(x) - 5$. Solve $f(x) = 0$.

Solution:

$$2\ln(x) - 5 = 0$$
$$2\ln(x) = 5$$
$$\ln(x) = 5/2$$
$$e^{\ln(x)} = e^{5/2}$$
$$x = e^{5/2}$$

(m) Let $f(x) = 3\ln(x+1) + 7$. Solve $f(x) = 0$.

Solution:

$$3\ln(x+1) + 7 = 0$$
$$3\ln(x+1) = -7$$
$$\ln(x+1) = -7/3$$
$$e^{\ln(x+1)} = e^{-7/3}$$
$$(x+1) = e^{-7/3}$$
$$x = e^{-7/3} - 1$$

3. Let $f(x) = x^2$ and $g(x) = \frac{1}{x} + 1$.

(a) Find $f(x) + g(x)$.

Solution:
$x^2 + \frac{1}{x} + 1$.

(b) Find $f(x)g(x)$.

Solution:
$x^2\left(\frac{1}{x} + 1\right)$.

(c) Find $\dfrac{f(x)}{g(x)}$.

Solution:
$$\frac{x^2}{\frac{1}{x}+1}.$$

(d) Find $f(g(x))$.

Solution:
$$f\left(\frac{1}{x}+1\right)=\left(\frac{1}{x}+1\right)^2.$$

(e) Find $g(f(x))$.

Solution:
$$g(x^2)=\frac{1}{x^2}+1.$$

Linear, Power Function, and Exponential Modelling

4. The two sections of Applied Calculus that I taught this semester started with 61 students. 15 weeks later I had 56 students.

   (a) Assuming that the students continued dropped the class at a linear rate, write an equation for S, the number of students, as a function of t, the number of weeks since the beginning of the semester.

   (b) According to your function, how many students did I have in week 5?

Solution: (a)

line through $(0, 61)$ and $(15, 56)$

$$\begin{aligned}
y &= m(x-x_0)+y_0\\
x_0 &= 0\\
y_0 &= 61\\
m &= \frac{56-61}{15-0}\\
&= \frac{-5}{15}\\
&= -\frac{1}{3}\\
y &= -\frac{1}{3}(x-0)+61\\
&= -\frac{1}{3}x+61
\end{aligned}$$

(b)

$$\begin{aligned}
y &= -\frac{1}{3}(5)+61\\
&= -\frac{5}{3}+61\\
&= 61-(1+\frac{2}{3})\\
&= 59.333
\end{aligned}$$

5. The arctic ice caps appears to be shrinking at a rate of 4% per decade[1].

   (a) Assuming that the rate of decrease of ice remains the same, write an equation for I, the percent of the current ice that will be left, as a function of t, the number of decades from now.

   (b) According to your equation, what percent of the current amount ice will be left in 50 years?

Solution: (a)

$$\begin{aligned} I &= Ca^t \quad C = \% \text{ of current ice, } t\text{=decades, } a = 1 + r,\ r = \% \text{ change} \\ &= 1 \cdot (1 - 0.04)^t \\ I &= (0.96)^t \end{aligned}$$

(b)

$$\begin{aligned} I &= (0.96)^5 \\ &= 0.815 \\ &= 81.5\% \text{ of current ice} \end{aligned}$$

Functions in Economics

6. (Hughes-Hallet, 4e, #9) A company that makes Adirondack chairs has fixed costs of \$5000 and variable costs of \$30 per chair. The company sells the chairs for \$50 each.

   (a) Find formulas for the cost and revenue functions.

   (b) Find the marginal cost and marginal revenue.

   (c) Graph the cost and revenue functions on the same axes.

   (d) Find the break-even point.

Solution:

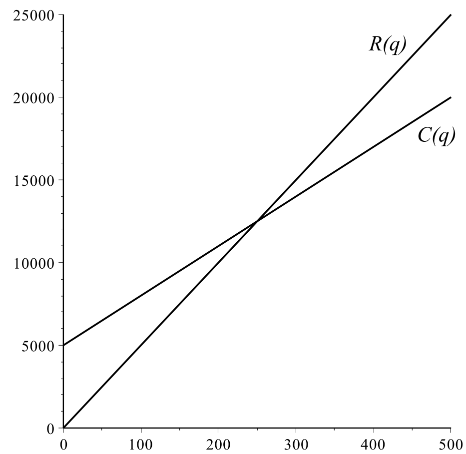
(a)

$$\begin{aligned} C(q) &= 5000 + 30q \\ R(q) &= 50q \end{aligned}$$

(b)

$$\begin{aligned} &MC = 30 \\ &MR50 \end{aligned}$$

[1] From `http://www.theguardian.com/environment/2013/sep/18/how-fast-is-arctic-sea-ice-melting`



(c)

(d)

$$\begin{aligned} C(q) &= R(q) \\ 5000 + 30q &= 50q \\ 20q &= 5000 \\ q &= \frac{5000}{20} \\ q &= 250 \end{aligned}$$

7. (Hughes-Hallett, 4e, #28) The demand curve for a product is given by $q = 120,000 - 500p$ and the supply curve is given by $q = 1000p$ for $0 \le q \le 120,000$, where price is in dollars.

   (a) At a price of $100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?

   (b) Find the equilibrium price of quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to the equilibrium price?

Solution:

(a)

$$\begin{aligned} \text{consumers willing to buy: } & q120,000 - 500p \\ &= 120,000 - 500(100) \\ &= 70,000 \\ \text{suppliers willing to make: } q &= 1000p \\ &= 1000(100) \\ &= 100,000 \end{aligned}$$

Market forces will move prices down, because there are too many items being made.

(b)

$$\begin{aligned} \text{supply} &= \text{demand} \\ 1000p &= 120000 - 500p \end{aligned}$$

$$500p = 120000$$
$$p = \frac{120000}{500}$$
$$= 80$$

This agrees with (a) because from $p = 100$ the price will move down to $p = 80$.

Approximating the Derivative

8. Let $f(x) = e^{-x^2}$. Find an approximation of $f'(2)$ by filling in the following table using the same rules as on the quiz[2].

Solution:

x	$f(x)$	$\frac{f(2)-f(x)}{2-x}$
1.9	0.02705185	−0.08736208
1.99	0.01906121	−0.07455692
1.999	0.01838903	−0.07339089
2	0.01831564	ERR
2.1	0.01215518	−0.06160461
2.01	0.01759571	−0.07199261
2.001	0.01824250	−0.07313447

The numbers we calculated for $x = 1.999$ and 2.001 are our best approximations. From them we can say

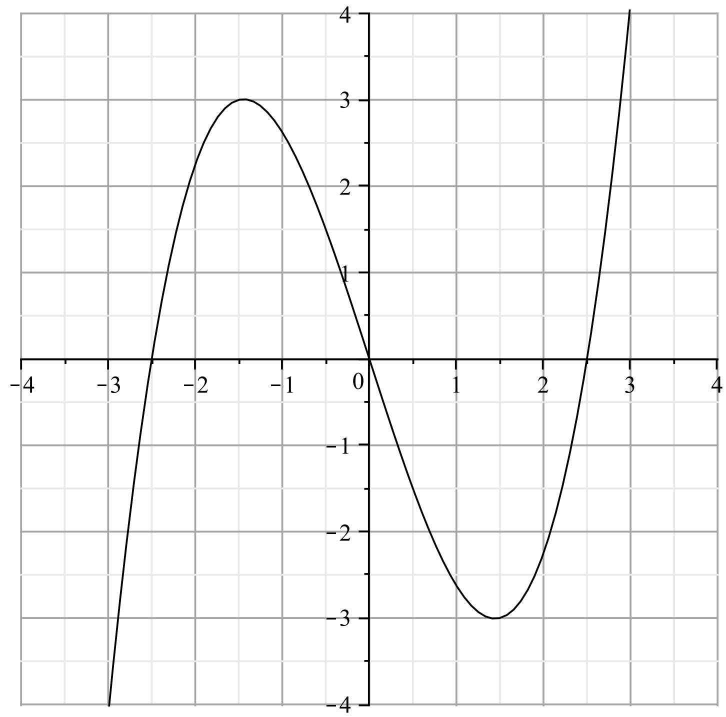
$$f'(2) \approx -0.073$$

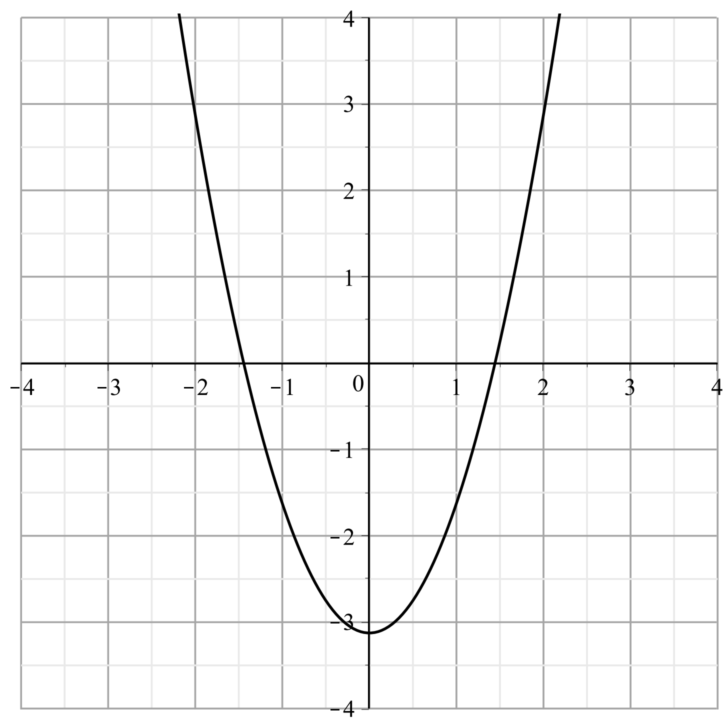
Graphing Derivatives

9. Shown below is the graph of a cubic function, i.e. a function of the form $y = ax^3 + bx^2 + cx + d$. Sketch a graph of the derivative; make sure that your sketch is positive/negative/zero in the right places, and that it has the right shape.

Solution:

[2] You should plug numbers in for x that are close to 2. They should be close enough that you get two approximations of $f'(2)$ that are the same to the *second* decimal place. You should always use at least 8 digits in every step of the calculation.





To see how to make the second graph, start with the points where f is flat:

graph of $f(x)$: $\quad m = 0$ at $x = -1.5$ and $x = 1.5$

This means that the graph of $f'(x)$ should have

$$\text{graph of } f'(x): \quad y = 0 \text{ at } x = -1.5 \text{ and } x = 1.5$$

Similarly, see where f is going up:

$$\text{graph of } f(x): \quad m \approx 2 \text{ at } x = -2 \text{ and } x = 2$$

This means that the graph of $f'(x)$ should have

$$\text{graph of } f'(x): \quad y = 2 \text{ at } x = -2 \text{ and } x = 2$$

Finally, see where f is going down:

$$\text{graph of } f(x): \quad m \approx -3 \text{ at } x = 0$$

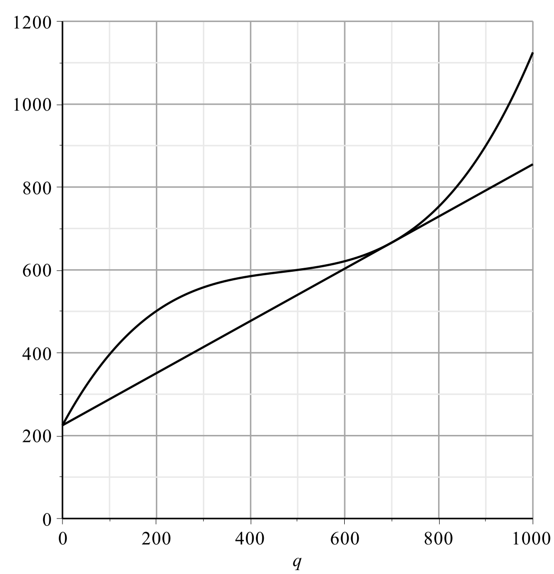
This means that the graph of $f'(x)$ should have

$$\text{graph of } f'(x): \quad y = -3 \text{ at } x = 0$$

Marginal Cost and Revenue

10. The graph below shows a total cost function. Estimate the marginal cost at $q = 700$.

Solution: We start by graphing a tangent line (simply move a straight edge towards the graph of $C(q)$ until it just barely touches the graph at $q = 700$)



Now we estimate the slope of the straight line. The best way to do this is to use points that are as far apart as possible, or to see if the line intersects exactly at one of the grid points.

I'll use points far apart:

$$(0, 210) \text{ and } (1000, 810)$$

appear to be on the line. Thus

$$MC = m \approx \frac{810 - 210}{1000 - 0} = \frac{600}{1000} = 3/5$$

11. A company is making complicated things with a cost function given by $C(q) = 7e^{-q} - 5\sqrt{q} + 11000q^2$ and revenue function given by $R(q) = 25000q$.

    (a) Find the marginal cost and marginal revenue.

    (b) At $q = 300$ is the profit increasing or decreasing?

Solution:

(a)

$$\begin{aligned} MC &= \frac{d}{dq}C(q) \\ &= -7e^{-q} - \frac{5}{2}q^{-1/2} + 22000q \\ MR &= \frac{d}{dq}R(q) \\ &= 25000 \end{aligned}$$

(b)

$$\begin{aligned} MC(300) &= -7e^{-300} - \frac{5}{2}(300)^{-1/2} + 22000(300) \\ &= 6599999.86 \\ MR(300) &= 25000 \end{aligned}$$

Profic is decreasing since $MC > MR$.

Basic derivatives

12. Find the following derivatives

    (a) $\frac{d}{dx}3x^6$

    Solution:
    $18x^5$

    (b) $\frac{d}{dx}5\sqrt[3]{x}$

    Solution:
    $\frac{5}{3}x^{-2/3}$ (note that $\sqrt[3]{x} = x^{1/3}$)

    (c) $\frac{d}{dx}\frac{10}{x^5}$

    Solution:
    $-50x^{-6}$ (note that $\frac{10}{x^5} = 10x^{-5}$)

(d) $\dfrac{d}{dx}17e^x$

Solution:
$17e^x$

(e) $\dfrac{d}{dx}17\ln(x)$

Solution:
$17 \cdot \dfrac{1}{x}$

Combinations of functions

13. Find the following derivatives

(a) $\dfrac{d}{dx}3(2x-5)^6$

Solution:

$$\frac{d}{dx}3\left(\boxed{2x-5}\right)^6 = 18\left(\boxed{2x-5}\right)^5 \cdot \boxed{2x-5}'$$
$$= 18(2x-5)^5 \cdot 2$$

(b) $\dfrac{d}{dx}5\sqrt[3]{-7x+11}$

Solution:

$$\frac{d}{dx}5\sqrt[3]{\boxed{-7x+11}} = \frac{5}{3}\left(\boxed{-7x+11}\right)^{-2/3} \cdot \boxed{-7x+11}'$$
$$= \frac{5}{3}(-7x+11)^{-2/3} \cdot (-7)$$

(c) $\dfrac{d}{dx}\dfrac{10}{(4x-5)^5}$

Solution:

$$\frac{d}{dx}\frac{10}{(\boxed{4x-5})^5} = -50\left(\boxed{4x-5}\right)^{-6} \cdot \boxed{4x-5}'$$
$$= -50\,(4x-5)^{-6} \cdot 4$$

(d) $\dfrac{d}{dx}17e^{-x+10}$

Solution:

$$\frac{d}{dx}17e^{-x+10}17e^{\boxed{-x+10}} \cdot \boxed{-x+10}'$$
$$17e^{-x+10} \cdot (-1)$$

(e) $\dfrac{d}{dx}17\ln(3x+5)$

Solution:

$$\frac{d}{dx}17\ln\left(\boxed{3x+5}\right) = 17\cdot\frac{1}{\boxed{3x+5}}\cdot\boxed{3x+5}'$$

14. Find the following derivatives

(a) $\dfrac{d}{dx}(3x^6+5\sqrt[3]{x})(\dfrac{10}{x^5}-17e^x)$

Solution:

$$\frac{d}{dx}\underbrace{(3x^6+5\sqrt[3]{x})}_{f}\underbrace{(\frac{10}{x^5}-17e^x)}_{g}$$

$$f' = 18x^5+\frac{5}{3}x^{-2/3}$$

$$g' = -50x^{-6}-17e^x$$

$$f'g+fg' = \left(18x^5+\frac{5}{3}x^{-2/3}\right)\left(\frac{10}{x^5}-17e^x\right)+(3x^6+5\sqrt[3]{x})\cdot\left(-50x^{-6}-17e^x\right)$$

(b) $\dfrac{d}{dx}\dfrac{2x^2-x}{e^x+x^2}$

$$\frac{2x^2-x \leftarrow f}{e^x+x^2 \leftarrow g}$$

$$f' = 4x-1$$

$$g' = e^x+2x$$

$$\frac{f'g-fg'}{g^2} = \frac{(4x-1)(e^x+x^2)-(2x^2-x)(e^x+2x)}{(e^x+x^2)^2}$$

Derivatives and Tangent Lines

15. (a) Find the Equation of tangent line at $x=5$ for $f(x)=\ln(x)$.

Solution:

$$ym(x-x_0)+y_0$$

$$x_0 = 5$$

$$y_0 = f(5) = \ln(5)$$

$$f'(x) = \frac{1}{x}$$

$$m = f'(5) = \frac{1}{5}$$

$$y = \frac{1}{5}(x-5)+\ln(5)$$

(b) Find the Equation of tangent line at $x = 9$ for $f(x) = 5\sqrt{x}$.

Solution:

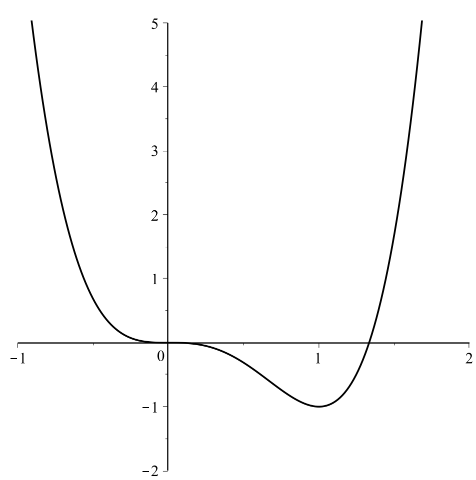
$$\begin{aligned} & ym(x - x_0) + y_0 \\ x_0 &= 9 \\ y_0 &= f(9) = 5\sqrt{9} \\ &= 15 \\ f'(x) &= \frac{5}{2}x^{-1/2} \\ m &= f'(9) = \frac{5}{2}(9)^{-1/2} \\ &= \frac{5}{2} \cdot \frac{1}{\sqrt{9}} \\ &= \frac{5}{2} \cdot \frac{1}{3} \\ &= \frac{5}{6} \\ y &= \frac{5}{6}(x - 9) + 15 \end{aligned}$$

Local Max/Mins

16. Let $f(x) = 3x^4 - 4x^3$.

(a) Using your calculator, graph $f(x)$. Use a window that shows all the "interesting" features (in particular it should show the local max/mins).

Solution:



Note: your window has to be small enough that I can see two things: (1) that the point at $(0, 0)$ is neither a max nor min, (2) that the point $(1, -1)$ is a local min.

(b) Describe the critical points, the local max and mins, and where the function is increasing/decreasing.

Solution:
critical points: $x = 0, 1$
$x = 1$ is l. min at $x = 1$.
$x = 0$ is neither
$f \uparrow$ to right of $x = 1$.
$f \downarrow$ to left of $x = 1$.

(c) Take the first derivative of $f(x)$, find the critical points algebraically.

$$\begin{aligned} f'(x) &= 12x^3 - 12x^2 \\ f'(x) &= 0 \\ 12x^3 - 12x^2 &= 0 \\ 12x^2(x-1) &= 0 \\ x^2 &= 0 \text{ or } (x-1) = 0 \\ x &= 0, 1 \end{aligned}$$

(d) Summarize your work in a "1D# table" (1st Derivative Number Line Table[3]) table that shows the first derivative test and the conclusions that it gives you.

Solution:

	neither		l.min	
	$x = 0$		$x = 1$	
$f \searrow$	|	$f \searrow$	|	$f \nearrow$
$f' < 0$	$f' = 0$	$f' < 0$	$f' = 0$	$f' > 0$

Global Max/Mins

17. Let $f(x) = x^{10} - 10x$.

(a) Find the critical points of $f(x)$.

Solution:

$$\begin{aligned} f'(x) &= 10x^9 - 10 \\ f'(x) &= 0 \\ 10x^9 - 10 &= 0 \\ 10(x^9 - 1) &= 0 \\ x^9 - 1 &= 0 \\ x^9 &= 1 \\ x &= \sqrt[9]{1} \\ x &= 1 \end{aligned}$$

[3]This should be a number line with the following information: you should label on the number line each critical point. Above each critical point you should indicate whether that point is a local max/min/neither. On top of the number line and between the critical points you should indicate whether f is increasing or decreasing. On the bottom of the number line, between the critical points and at each critical point, you should indicated whether f' is $+$, $-$ or 0.

(b) Apply the Global Max/Min test to find the absolute max/min on the interval $0 \le x \le 2$.

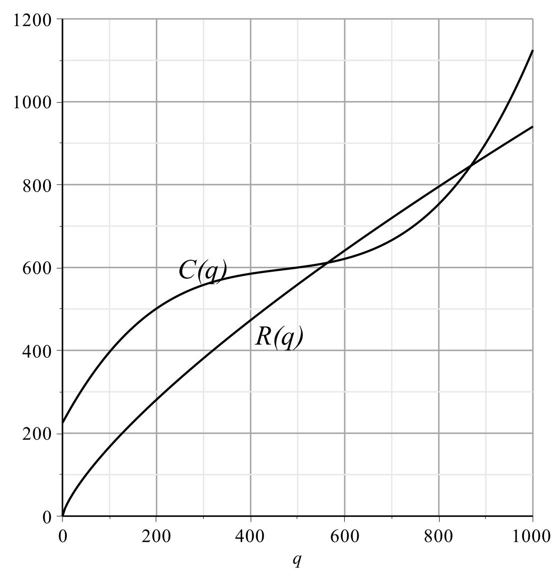
Solution:

x	y
1	-9
0	0
2	1004

G. max: $x = 2$, $y = 1004$
G. min: $x = 1$, $y = -9$

Optimizing Cost and Revenue

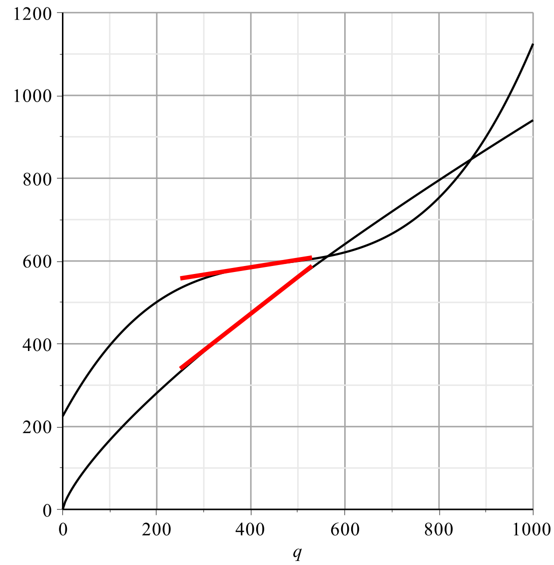
18. Shown below is a graph of cost and revenue for a certain company:



(a) If the production level is $q = 400$, should the company increase production to $q = 401$? Why or why not?

Solution:

Here's what the graphs look like with a small part of the tangent lines shown:



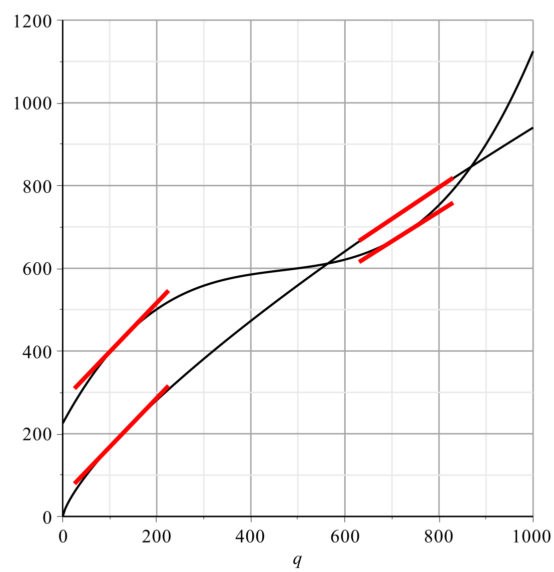
It should be clear that the slope of C is less than the slope of R. In other words:

$$MR > MC \Rightarrow \text{profit increases with } q$$

so they should increase production.

(b) Find two critical points of profit. Which one has maximum profit? Why?

Solution:
We find two points where the slopes are the same for C and R: these are at $q = 126$ and $q = 730$:



The one that is maximum profit is $q = 730$. The best way to tell this is just to the left of 730 we have $MR > MC$ and just to the right we have $MR < MC$.

19. A company has a cost function given by $C(q) = 10q + 1$ and revenue function given by $R(q) = 15\sqrt{q}$.

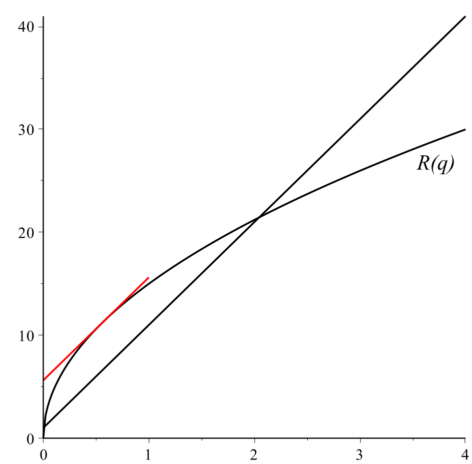
(a) Find the critical point of profit.

Solution:

$$\begin{aligned} MC &= 10 \\ MR &= \frac{15}{2}q^{-1/2} \\ MC &= MR \\ 10 &= \frac{15}{2}q^{-1/2} \\ 10 &= \frac{15}{2} \cdot \frac{1}{\sqrt{q}} \\ 10 &= \frac{15}{2\sqrt{q}} \\ 20\sqrt{q} &= 15 \text{ (cross multiply)} \\ \sqrt{q} &= \frac{15}{20} \\ \sqrt{q} &= 3/4 \\ q &= (3/4)^2 \\ q &= 9/16 \approx 0.562 \end{aligned}$$

(b) Is the critical point a maximum for profit? Why?

Solution:
The critical point is a maximum for profit. The best way to see this is to note that just to the left we have $MR > MC$ and just to the right we have $MR < MC$:



Average Cost

20. A company is making simple things with a cost function given by $C(q) = 5q^2 + 1$.

(a) Find the average cost at $q = 3$.

Solution:

$$\begin{aligned} a(q) &= \frac{C(q)}{q} \\ &= \frac{5q^2+1}{q} \\ a(3) &= \frac{5(3)^2+1}{3} \\ &= \frac{46}{3} \approx 15.333 \end{aligned}$$

(b) Is the average cost increasing or decreasing at $q = 3$?

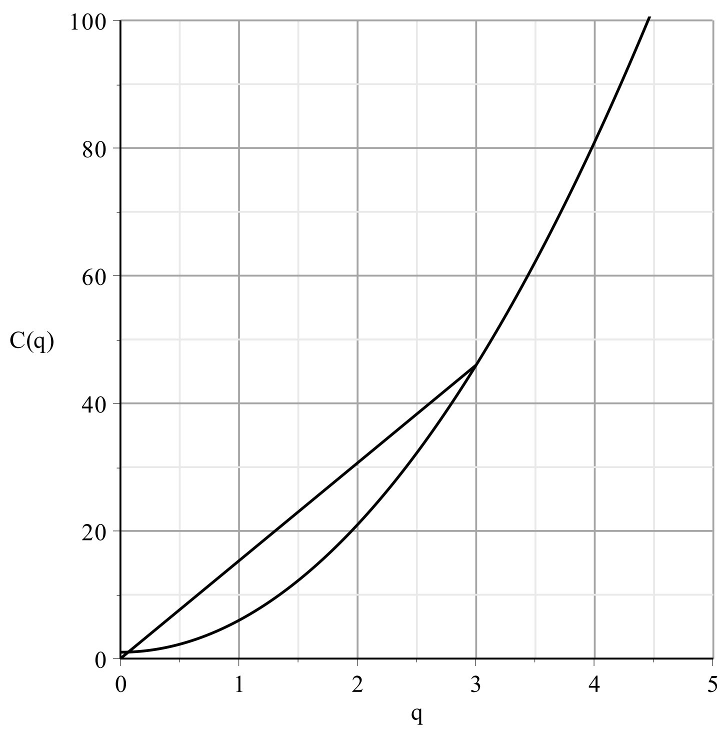
Solution:

$$\begin{aligned} MC &= 10q \\ MC(3) &= 30 \end{aligned}$$

The average cost is increasing because $MC(3) > a(3)$.

(c) Repeat the previous two parts graphically, using the graph of $5q^2 + 1$:

Solution:
The average cost equals the slope of the straight line shown below:



$$\begin{aligned} a(3) &= \text{slope of line} \\ &= \frac{\text{rise}}{\text{run}} \\ &= \frac{45}{3} \end{aligned}$$

If we look at $q = 3$ where the straight line intersects the curve, it's clear that the curve is steeper at that point. Thus, $MC(3) > a(3)$ and so the average cost is increasing.

Elasticity of Demand

21. You are given a demand function $q = 1000 - 10p^2$. (Note: I changed the formula for q to be more realistic.)

(a) Find the elasticity at $p = 5$.

Solution:

$$\begin{aligned} E &= \left|\frac{p}{q} \cdot \frac{dq}{dp}\right| \\ p &= 5 \\ q &= 1000 - 10(5)^2 \\ &= 1000 - 250 \\ &= 750 \\ \frac{dq}{dp} &= -20p \\ \left.\frac{dq}{dp}\right|_{p=5} &= -20(5) \\ &= -100 \\ E &= \left|\frac{5}{750} \cdot (-100)\right| \\ &= \left|\frac{-500}{750}\right| \\ &= \left|-\frac{2}{3}\right| \\ &= \frac{2}{3} \approx 0.666 \end{aligned}$$

(b) If we increase price, do we expect R to increase or decrease?

Solution:
We expect R to increase because $E < 1$.

Interpreting Integrals as Velocity and Area

22. Set up an integral to find the area below $y = e^{-x^2}$, above the x-axis, and between 0 and 1.

Solution:

$$\int_0^1 e^{-x^2}\,dx$$

23. Set up an integral to find the net distance travelled by a falling object with velocity given by $v(t) = -9.8t + 12$, from $t = 1$ to $t = 5$.

Solution:

$$\int_1^5 -9.8t + 12\,dt$$

Calculating using tables, graphs and calculators

24. Calculate $\int_1^{10} f(x)\,dx$ where $f(x)$ is given by a table of numbers:

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	5.3	2.9	-1.1	3.1	1.5	2.7	3.1	2.9	1.8	1.5

Solution:

$$\begin{aligned}\int_1^{10} f(x)\,dx &\approx 5.3\times 1 + 2.9\times 1 + -1.1\times 1 + 3.1\times 1 + 1.5\times 1\\ &\quad + 2.7\times 1 + 3.1\times 1 + 2.9\times 1 + 1.8\times 1\\ &= 22.2\end{aligned}$$

$$\text{OR}\quad \begin{aligned}\int_1^{10} f(x)\,dx &\approx 2.9\times 1 + -1.1\times 1 + 3.1\times 1 + 1.5\times 1 + 2.7\times 1\\ &\quad + 3.1\times 1 + 2.9\times 1 + 1.8\times 1 + 1.5\times 1\\ &= 18.4\end{aligned}$$

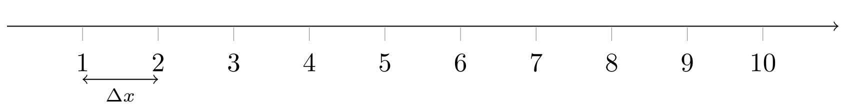
Note: it is *required* that you first write out the sum as I have done, before you calculate the total.

25. Calculate $\int_1^{10} x^2\,dx$ with a Left Hand Riemann Sum and $n = 9$.

Solution:

We start with the number line

Now we divide into 9 equal pieces:



So the x-values are 1, 2, 3, $\ldots$, 10.

For the left hand rule we use 1, 2,$\ldots$, 9:

$$\begin{aligned}\int_1^{10} x^2\,dx &= f(1)\times 1 + f(2)\times 1 + f(3)\times 1 + f(4)\times 1 + f(5)\times 1\\ &\quad + f(6)\times 1 + f(7)\times (1) + f(8)\times 1 + f(9)\times 1\\ &= (1)^2\times 1 + (2)^2\times 1 + (3)^2\times 1 + (4)^2\times 1 + (5)^2\times 1\\ &\quad + (6)^2\times 1 + (7)^2\times (1) + (8)^2\times 1 + (9)^2\times 1\\ &= 285\end{aligned}$$

For the right hand rule we use 2, 2,$\ldots$, 10:

$$\begin{aligned}\int_1^{10} x^2\,dx &= f(2)\times 1 + f(3)\times 1 + f(4)\times 1 + f(5)\times 1 + f(6)\times 1\\ &\quad + f(7)\times (1) + f(8)\times 1 + f(9)\times 1 + f(10)\times 1\\ &= (2)^2\times 1 + (3)^2\times 1 + (4)^2\times 1 + (5)^2\times 1 + (6)^2\times 1\\ &\quad + (7)^2\times (1) + (8)^2\times 1 + (9)^2\times 1 + (10)^2\times 1\\ &= 384\end{aligned}$$

Note: it is *required* that you first write out the sum in at least one of the ways I've done (i.e. either with "$f(\ )$" or with "$(\ )^2$") before you calculate the total.

26. Calculate $\displaystyle\int_0^1 e^{-x^2}\,dx$ using your calculator.

Solution:

You can enter

`MATH` `9: fnInt(` `e∧(-X²),X,0,1)` `Enter`

or

`Y=` `Y1=` `e∧(-X²)` `WINDOW` `xmin=0` `xmax=1` `2cnd` `CALC` `7: ∫f(x)dx` `Lower Limit? 0` `Upper Limit? 1`

and you should get

$$0.7468$$