Final Exam. MA 151, Applied Calculus, Spring 2014

Your name: _____

Your student number: _____

Showing work. Each answer must include step-by-step work on your page. Even calculations which will be done in a calculator, should first appear as formulas on the page (here, "formulas" can mean function expressions, like $C(50) - C(49)$ or $\ln(0.1) + \ln(0.2)$ etc.)

Time management. Probably not everyone will have enough time to do every problem correctly. I think it is better, and that you will get a better score, if you (1) skip the hardest problems until later, (2) work carefully and write more complete steps so that you don't make mistakes and you get better partial credit, and you can see which part of your work is correct, etc.

Problem	Possible points	Points received
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total	140	

Please sign the following pledge:

On my honor I have neither given nor received any aid on this exam; I have upheld the ideals of the honor code.

Signature _____

Chapter 1: Functions and Applications

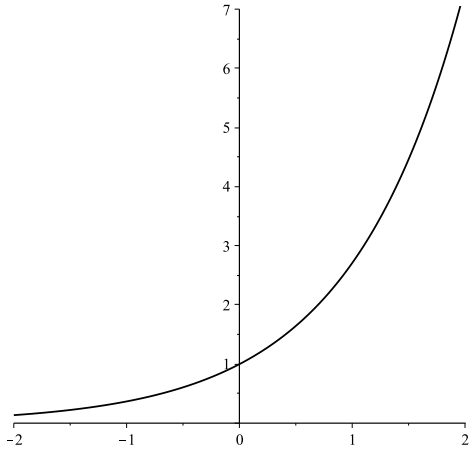
Basic Functions

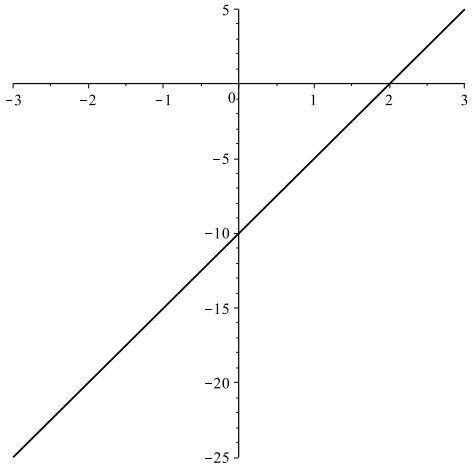
1. Identify each of the graphs below as one of the following functions:

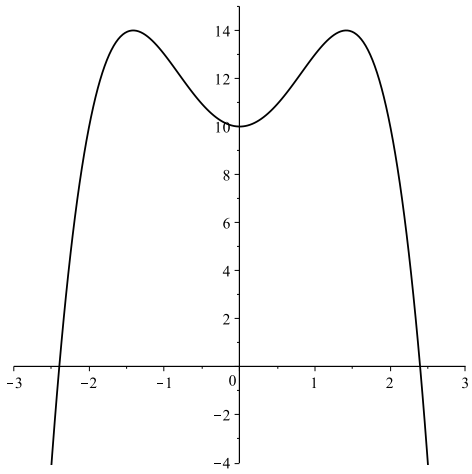
(a) $x^3 - 4x^2 + x$ (d) x^2 (g) $(1/2)^x$

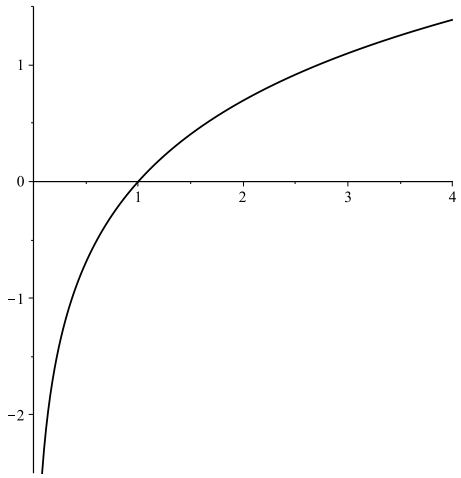
(b) e^x (e) $-5x + 10$ (h) $-x^4 + 4x^2 + 10$

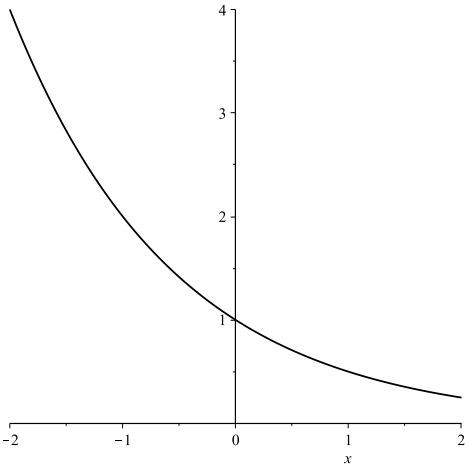
(c) $\ln(x)$ (f) x^3 (i) $5x - 10$







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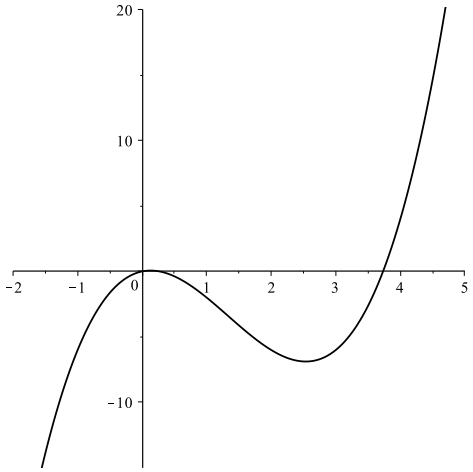
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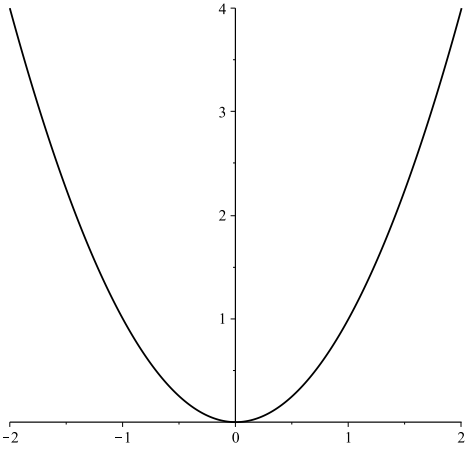
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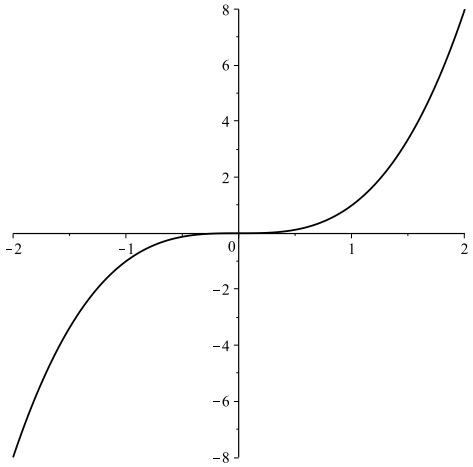
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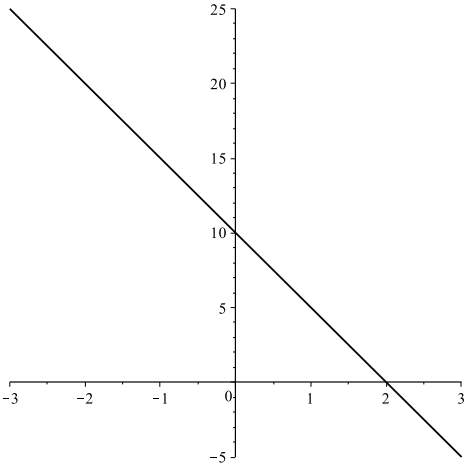
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Function notation, solving function equations

2. (This problem as a whole is obviously too long for the final: but I could have a problem with a couple of parts like this.)
   (a) Let $f(x) = 3x + 5$. Solve $f(x) = 3$.
   (b) Let $f(x) = x^2$. Solve $f(x) = 5$.
   (c) Let $f(x) = 2x^3 - 5$. Solve $f(x) = -10$.
   (d) Let $f(x) = x^2 - 2x - 15$. Solve $f(x) = 0$.
   (e) Let $f(x) = x^2 - 2x - 14$. Solve $f(x) = 0$.
   (f) Let $f(x) = e^x - 2x^2e^x$. Solve $f(x) = 0$.
   (g) Let $f(x) = \dfrac{1}{x} + 1$. Solve $f(x) = 0$.
   (h) Let $f(x) = \dfrac{2}{x^2} - 5$. Solve $f(x) = 0$.
   (i) Let $f(x) = 3e^x - 5$. Solve $f(x) = 0$.
   (j) Let $f(x) = 4e^{2x} - 3$. Solve $f(x) = 0$.
   (k) Let $f(x) = \ln(x) + 5$. Solve $f(x) = 0$.
   (l) Let $f(x) = 2\ln(x) - 5$. Solve $f(x) = 0$.
   (m) Let $f(x) = 3\ln(x + 1) + 7$. Solve $f(x) = 0$.

3. Let $f(x) = x^2$ and $g(x) = \dfrac{1}{x} + 1$.
   (a) Find $f(x) + g(x)$.
   (b) Find $f(x)g(x)$.
   (c) Find $\dfrac{f(x)}{g(x)}$.
   (d) Find $f(g(x))$.
   (e) Find $g(f(x))$.

Linear, Power Function, and Exponential Modelling

4. The two sections of Applied Calculus that I taught this semester started with 61 students. 15 weeks later I had 56 students.
   (a) Assuming that the students continued dropped the class at a linear rate, write an equation for S, the number of students, as a function of t, the number of weeks since the beginning of the semester.
   (b) According to your function, how many students did I have in week 5?

5. The arctic ice caps appears to be shrinking at a rate of 4% per decade[1].
   (a) Assuming that the rate of decrease of ice remains the same, write an equation for I, the percent of the current ice that will be left, as a function of t, the number of decades from now.
   (b) According to your equation, what percent of the current amount ice will be left in 50 years?

[1] From `http://www.theguardian.com/environment/2013/sep/18/how-fast-is-arctic-sea-ice-melting`

Functions in Economics

6. (Hughes-Hallet, 4e, #9) A compny that makes Adirondack chairs has fixed costs of $5000 and variable costs of $30 per chair. The company sells the chairs for $50 each.
   (a) Find formulas for the cost and revenue functions.
   (b) Find the marginal cost and marginal revenue.
   (c) Graph the cost and revenue functions on the same axes.
   (d) Find the break-even point.
7. (Hughes-Hallett, 4e, #28) The demand curve for a product is given by $q = 120,000 - 500p$ and the supply curve is given by $q = 1000p$ for $0 \leq q \leq 120,000$, where price is in dollars.
   (a) At a price of $100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?
   (b) Find the equilibrium price of quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to the equilibrium price?

Chapter 2: The derivative

Approximating the Derivative

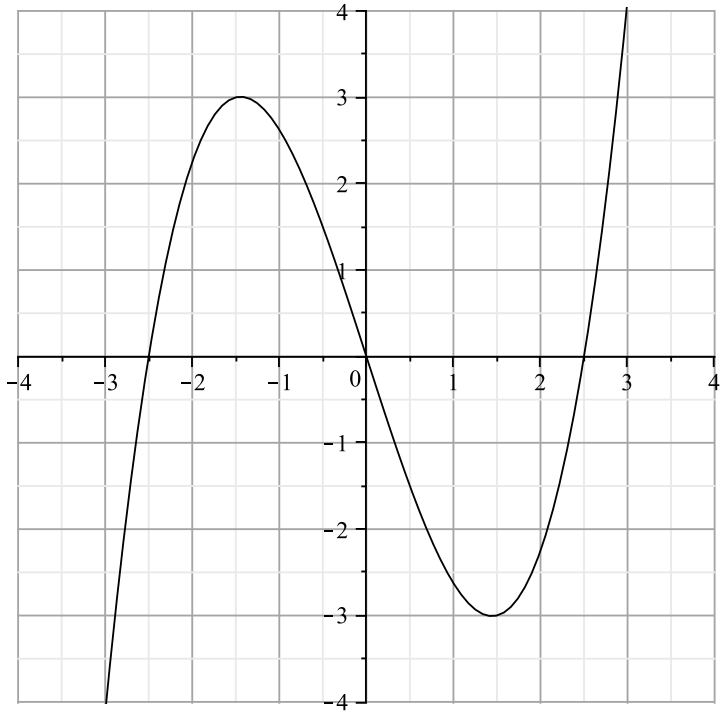
8. Let $f(x) = e^{-x^2}$. Find an approximation of $f'(2)$ by filling in the following table using the same rules as on the quiz[1].

x	$f(x)$	$\dfrac{f(2) - f(x)}{2 - x}$

[1] You should plug numbers in for x that are close to 2. They should be close enough that you get two approximations of $f'(2)$ that are the same to the first decimal place. You should always use at least 8 digits in every step of the calculation.

Graphing Derivatives

9. Shown below is the graph of a cubic function, i.e. a function of the form $y = ax^3 + bx^2 + cx + d$. Sketch a graph of the derivative; make sure that your sketch is positive/negative/zero in the right places, and that it has the right shape.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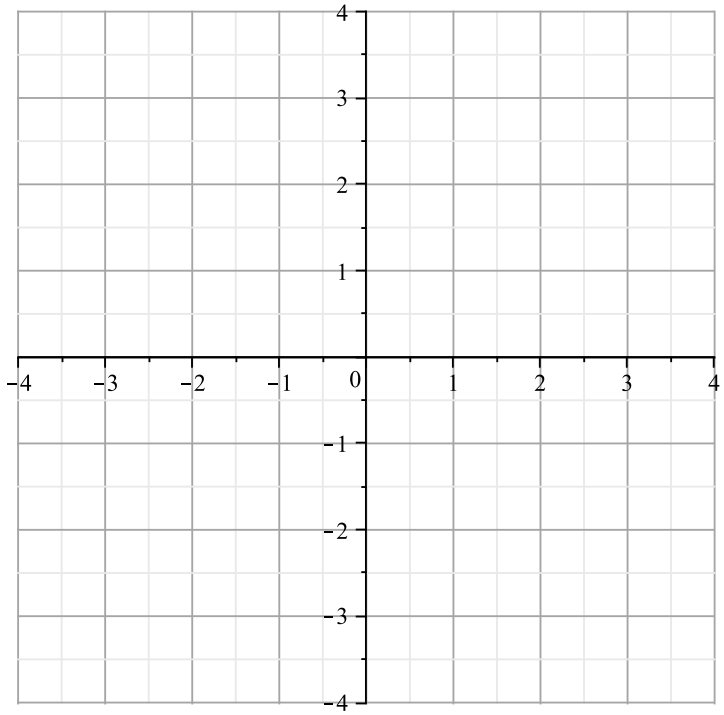
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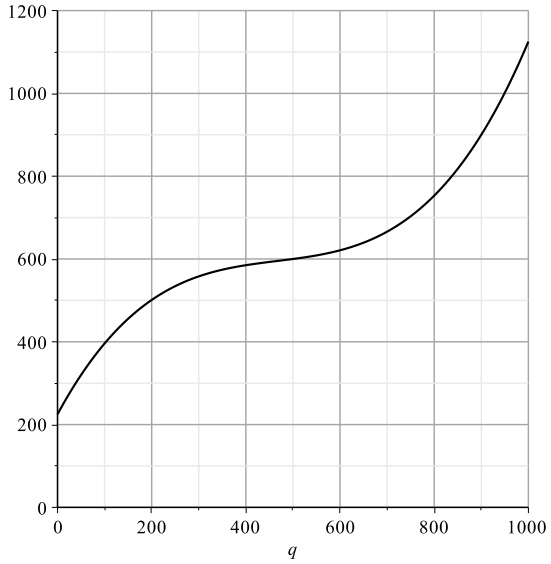
Marginal Cost and Revenue

10. The graph below shows a total cost function. Estimate the marginal cost at $q = 700$.



11. A company is making complicated things with a cost function given by $C(q) = 7e^{-q} - 5\sqrt{q} + 11000q^2$ and revenue function given by $R(q) = 25000q$.

    (a) Find the marginal cost and marginal revenue.

    (b) At $q = 300$ is the profit increasing or decreasing?

Chapter 3: Rules for Derivatives

Basic derivatives

12. Find the following derivatives

    (a) $\dfrac{d}{dx}3x^6$

    (b) $\dfrac{d}{dx}5\sqrt[3]{x}$

    (c) $\dfrac{d}{dx}\dfrac{10}{x^5}$

    (d) $\dfrac{d}{dx}17e^x$

    (e) $\dfrac{d}{dx}17\ln(x)$

Combinations of functions

13. Find the following derivatives

    (a) $\frac{d}{dx}3(2x-5)^6$

    (b) $\frac{d}{dx}5\sqrt[3]{-7x+11}$

    (c) $\frac{d}{dx}\frac{10}{(4x-5)^5}$

    (d) $\frac{d}{dx}17e^{-x+10}$

    (e) $\frac{d}{dx}17\ln(3x+5)$

14. Find the following derivatives

    (a) $\frac{d}{dx}(3x^6+5\sqrt[3]{x})(\frac{10}{x^5}-17e^x)$

    (b) $\frac{d}{dx}\frac{2x^2-x}{e^x+x^2}$

Derivatives and Tangent Lines

15. (a) Find the Equation of tangent line at $x=5$ for $f(x)=\ln(x)$.

    (b) Find the Equation of tangent line at $x=9$ for $f(x)=5\sqrt{x}$.

Chapter 4: Using the Derivative

Local Max/Mins

16. Let $f(x)=3x^4-4x^3$.

    (a) Using your calculator, graph $f(x)$. Use a window that shows all the "interesting" features (in particular it should show the local max/mins).

    (b) Describe the critical points, the local max and mins, and where the function is increasing/decreasing.

    (c) Take the first derivative of $f(x)$, find the critical points algebraically.

    (d) Summarize your work in a "1D# table" (1st Derivative Number Line Table[1]) table that shows the first derivative test and the conclusions that it gives you.

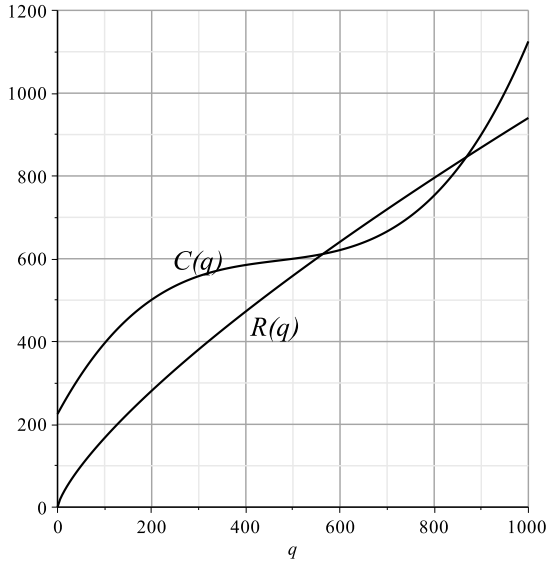
[1]This should be a number line with the following information: you should label on the number line each critical point. Above each critical point you should indicate whether that point is a local max/min/neither. On top of the number line and between the critical points you should indicate whether f is increasing or decreasing. On the bottom of the number line, between the critical points and at each critical point, you should indicated whether f' is $+$, $-$ or 0.

Global Max/Mins

17. Let $f(x) = x^{10} - 10x$.
    (a) Find the critical points of $f(x)$.
    (b) Apply the Absolute Max/Min test to find the absolute max/min on the interval $0 \leq x \leq 2$.

Optimizing Cost and Revenue

18. Shown below is a graph of cost and revenue for a certain company:

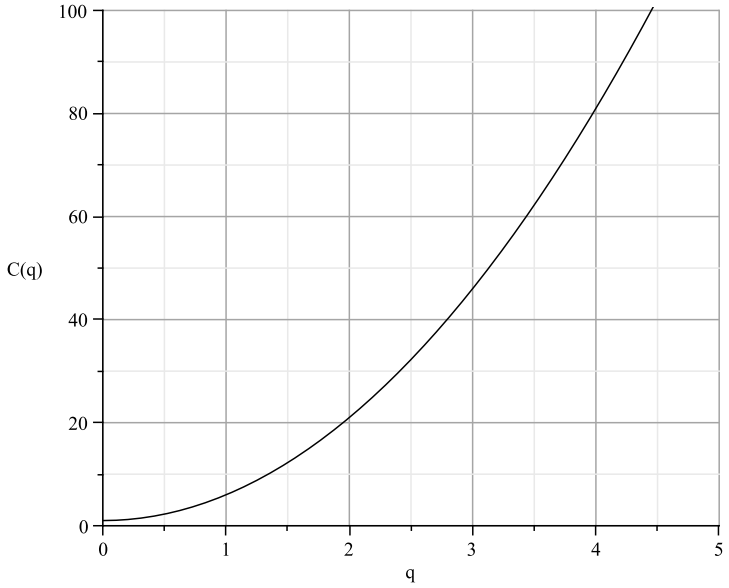


    (a) If the production level is $q = 400$, should the company increase production to $q = 401$? Why or why not?
    (b) Find two critical points of profit. Which one has maximum profit? Why?

19. A company has a cost function given by $C(q) = 10q + 1$ and revenue function given by $R(q) = 15\sqrt{q}$.
    (a) Find the critical point of profit.
    (b) Is the critical point a maximium for profit? Why?

Average Cost

20. A company is making simple things with a cost function given by $C(q) = 5q^2 + 1$.
    (a) Find the average cost at $q = 3$.
    (b) Is the average cost increasing or decreasing at $q = 3$?
    (c) Repeat the previous two parts graphically, using the graph of $5q^2 + 1$:



Elasticity of Demand

21. You are given a demand function $q = 100 - 10p^2$.
    (a) Find the elasticity at $p = 5$.
    (b) If we increase price, do we expect R to increase or decrease?

Chapter 5: Definite Integrals

Interpreting Integrals as Velocity and Area

22. Set up an integral to find the area below $y = e^{-x^2}$, above the x-axis, and between 0 and 1.

23. Set up an integral to find the net distance travelled by a falling object with velocity given by $v(t) = -9.8t + 12$, from $t = 1$ to $t = 5$.

Calculating using tables, graphs and calculators

24. Calculate $\int_1^{10} f(x)\,dx$ where $f(x)$ is given by a table of numbers:

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	5.3	2.9	-1.1	3.1	1.5	2.7	3.1	2.9	1.8	1.5

25. Calculate $\int_1^{10} x^2\,dx$ with a Left Hand Riemann Sum and $n = 9$.

26. Calculate $\int_0^1 e^{-x^2}\,dx$ using your calculator.