Disk/Washer Method $\int_a^b A(x) \, dx$ or $\int_a^b A(y) \, dy$

Take cross-sections PERPENDICULAR to axis of revolution.
If cross-section is a solid disk, $A = \pi R^2$
If cross-section is a washer/ring/annulus, $A = \pi R^2 - \pi r^2$

**Axis of Revolution is HORIZONTAL:** integrate with respect to $x$:

\begin{align*}
V &= \int_a^b \pi [f(x)]^2 \, dx \\
V &= \int_a^b \pi [f(x)]^2 - \pi [g(x)]^2 \, dx
\end{align*}

Examples of regions that can be done with either the disk/washer method or the shell method: see §6.2, #19-30.
Disk/Washer Method (cont.) \[ \int_{a}^{b} A(x) \, dx \text{ or } \int_{a}^{b} A(y) \, dy \]

Take cross-sections PERPENDICULAR to axis of revolution.
If cross-section is a solid disk, \( A = \pi R^2 \)
If cross-section is a washer/ring/annulus, \( A = \pi R^2 - \pi r^2 \)

**Axis of Revolution is VERTICAL:** integrate with respect to \( y \):

\[
V = \int_{a}^{b} \pi [f(y)]^2 \, dy
\]

**Axis of Revolution is HORIZONTAL:** integrate with respect to \( x \):

\[
V = \int_{a}^{b} \pi [f(x)]^2 \, dx
\]

Examples of regions that are best to use the disk/washer method:

- \( y = 1/x, \ x = 1, \ x = 2, \ y = 0 \) about the \( x \)-axis, or about the lines \( y = -1, \ y = 5 \)
- \( y = \cos x, \ y = \sin x, \ x = 0, \ x = \pi/6 \) about the \( x \)-axis, or about the lines \( y = 1, \ y = -1 \)
- \( x = 2y - y^2, \ x = 0, \) about the \( y \)-axis, or about the lines \( x = 5, \ x = -5 \)
Shell Method $\int_a^b 2\pi Rh \, dx$ or $\int_a^b 2\pi Rh \, dy$

Take cross-sections PARALLEL to axis of revolution.
Figure out the radius $R$ from cross-section to the axis of revolution
Figure out the height $h$ of the cross-section

**Axis of Revolution is VERTICAL:** integrate with respect to $x$:

\[ V = \int_a^b 2\pi x f(x) \, dx \]

\[ V = \int_a^b 2\pi x [f(x) - g(x)] \, dx \]

\[ V = \int_a^b 2\pi (x+c) f(x) \, dx \]

\[ V = \int_a^b 2\pi (x+c)(f(x) - g(x)) \, dx \]

\[ V = \int_a^b 2\pi (c-x) f(x) \, dx \]

\[ V = \int_a^b 2\pi (c-x)(f(x) - g(x)) \, dx \]

Examples of regions that are best to use the shell method:
$y = 1/x, \ x = 1, \ x = 2, \ y = 0$ about the $y$-axis, or about the lines $x = 3, \ x = 0.5$
$y = \cos x, \ y = \sin x, \ x = 0, \ x = \pi/6$ about the $y$-axis, or about the lines $x = 2, \ x = -2$
$x = 2y - y^2, \ x = 0$, about the $x$-axis, or about the lines $y = 5, \ y = -5$
Shell Method (cont.) \[ \int_{a}^{b} 2\pi Rh \, dx \text{ or } \int_{a}^{b} 2\pi Rh \, dy \]

Take cross-sections PARALLEL to axis of revolution.  
Figure out the radius \( R \) from cross-section to the axis of revolution  
Figure out the height \( h \) of the cross-section  

**Axis of Revolution is HORIZONTAL:** integrate with respect to \( y \):

\[
V = \int_{a}^{b} 2\pi y f(y) \, dy
\]

\[
V = \int_{a}^{b} 2\pi y[f(y) - g(y)] \, dy
\]

\[
V = \int_{a}^{b} 2\pi(y + c)f(y) \, dy
\]

\[
V = \int_{a}^{b} 2\pi(y + c)(f(y) - g(y)) \, dy
\]

\[
V = \int_{a}^{b} 2\pi(c - y)f(y) \, dy
\]

\[
V = \int_{a}^{b} 2\pi(c - y)(f(y) - g(y)) \, dy
\]