

Math 490 - Homework 5

Due October 13, 2005

#1. Consider the topological space where the underlying set is $X = \mathbb{R}$ and the open sets come from the usual distance function on \mathbb{R} . Find an infinite family of subsets $A_1, A_2, \dots \subseteq \mathbb{R}$ which has the following two properties:

- a) A_n is also a neighborhood of 0 for each n .
- b) The infinite intersection $\bigcap_n A_n$ is **not** a neighborhood of 0.

Note: In theorem 3.1 we proved that the intersection of two neighborhoods of a point x is a neighborhood of x . This problem shows that this property does not extend to infinite intersections.

#2. Do problem 1 on page 86 of the textbook.

#3. Do problem 3 on page 86 of the textbook.

#4. Do problem 5 on page 86 of the textbook.

#5. Do problem 8 on page 87 of the textbook.