1. Do problem 19.1 on page 92 of the textbook.

2. In April 1975 Martin Gardner, the author of the Mathematical Recreations column in the magazine *Scientific American* published the map below as an April Fool’s Day joke, claiming that it could not be colored with fewer than five colors. Show that this really was a joke by finding a way to color the faces of the map using only four colors.

![Map Diagram]

3. In class we proved a theorem stating that a map is $k$-colorable(f) if and only if the dual graph is $k$-colorable(v). In the figure below, a map $G$ is given and a 4-coloring of the map is provided for you. (Note that the infinite face is colored green.)
   a) Prove that $\chi(G) = 4$. **Note:** Since you are given a 4-coloring of the faces, you know that $\chi(G) \leq 4$. In order to complete the proof, you need to show that it is impossible to color the faces of this map with fewer than 4 colors.
   b) Find the dual graph of $G$, and show how the given face-coloring of $G$ corresponds to a vertex-coloring of $G^*$. 

![Dual Graph Diagram]
4. Do problem 20.2 on page 95 of the textbook.

5. Do part (i) of problem 20.8 on page 96 of the textbook.

6. A cubic map is given in the picture below.
   a) Find a 3-coloring of the edges of this map.
   b) Use the method in the proof of Theorem 20.3 to find a 4-coloring of the faces of this graph.