Fibonacci Set 1
MTH 200

Here are some initial questions about Fibonacci numbers for you to think about. For problems 1-3, you should definitely be able to find the pattern, but it may be hard to actually prove that it is true. On the other hand, it is possible to find proofs for problems 4 and 5 (and 6 and 7 as they relate to 4 and 5) without having a lot of background.

1.a) For which values of n is f_n a multiple of 2?
b) For which values of n is f_n a multiple of 3?
c) For which values of n is f_n a multiple of 5?
d) For which values of n is f_n a multiple of 8?
e) Let f_k be a Fibonacci number. For which values of n is f_n a multiple of f_k?

2. Are there similar patterns for multiples of other numbers?

3. For which values of n can f_n be a prime number?

4.a) Find a formula for $f_1 + f_2 + f_3 + \cdots + f_n$ in terms of other Fibonacci numbers.
b) What about a formula for $f_1 + f_3 + f_5 + \cdots + f_{2n-1}$?
c) What about a formula for $f_2 + f_4 + f_6 + \cdots + f_{2n}$?
d) How about a formula for $f_1^2 + f_2^2 + f_3^2 + \cdots + f_n^2$? (This one's a little trickier than the others. Think about multiplication.)

5. Look at the series $\sum_{n=0}^{\infty} \frac{1}{f_n^2}$. Does this series converge or diverge?

6. The **Lucas Numbers** are defined by $L_1 = 1, L_2 = 3$ and $L_n = L_{n-1} + L_{n-2}$ for all $n \geq 3$. Are your answers for problems 1-4 also true for the Lucas numbers?

7. Define a **Generalized Fibonacci Sequence** by setting $G_1 = p$ and $G_2 = q$, where p and q are any integers, and then let $G_n = G_{n-1} + G_{n-2}$ for all $n \geq 3$. Are your answers for problems 1-4 always true for these sequences? (You can test it by picking specific numbers for p and q.)

8. Can you find any other similar properties of Fibonacci numbers that you think might be true? Can you prove them? Even if you can't prove something, bring it up in class next Friday.