Homework 8  
Math 162Q - Fall 2002  
Due November 11, 2002  

§10.2 #33, 35, 36, 37, 38a.  
§10.3 #2, 6, 8, 16, 17, 18.  

Quest Problems:  
#1. Problem 41 on page 654 of the textbook.  
#2. Problem 42 on page 654 of the textbook.  
#3. As we’ve seen in class, the equation  
\[(x(t), y(t)) = (\cos t, \sin t)\]  
parametrizes the unit circle (defined by \(x^2 + y^2 = 1\)) in the \(xy\)-plane. It does so in such a way that the angle between the \(x\)-axis and the line segment from the origin to \((x(t), y(t))\) is \(t\) radians. Another way of saying this is that the region bounded by the line segment, the \(x\)-axis, and the circle has area equal to \(\pi\) (the total area of the circle) times \(\frac{t}{2\pi}\) (the ratio of the size of the region to the size of the circle), or simply \(t/2\).

Now, define the hyperbolic sine and hyperbolic cosine functions by  
\[
\sinh(t) = \frac{e^t - e^{-t}}{2} \quad \text{and} \quad \cosh(t) = \frac{e^t + e^{-t}}{2}.
\]

This problem will ask you to show that the equation \((x(t), y(t)) = (\cosh t, \sinh t)\) parametrizes a familiar hyperbola in a way analogous to that described above.

a) Show that for all \(t\), the point \((\cosh t, \sinh t)\) lies on the hyperbola defined by \(x^2 - y^2 = 1\).

b) Sketch the above hyperbola and indicate which of its points are of the form \((x(t), y(t)) = (\cosh t, \sinh t)\).

c) Let \(R\) be the region enclosed by the \(x\)-axis, the hyperbola, and the line segment from the origin to the point \((\cosh t, \sinh t)\). Show that the area of \(R\) is \(t/2\) (just like in the circle case).