Homework 7
Math 162Q - Fall 2002
Due October 28, 2002

§7.5 #3, 15, 18, 22, 39, 44, 64, 74.
§7.8 #2, 3, 6, 10, 12, 17, 20, 32, 40, 50, 52, 53, 56, 61.
Note: Some of the integrals from section 7.5 may be tricky.
Note: Problem 17 of section 7.8 is misprinted in the book. The integral should be

$$\int_0^\infty \cos x \, dx.$$

Quest Problems:
#1. Define the function $\Gamma(x)$ (for $x \geq 1$) by the improper integral

$$\Gamma(x) = \int_0^\infty e^{-t} \, t^{x-1} \, dt.$$

a) What is $\Gamma(1)$?

b) Using the method of integration by parts, show that $\Gamma(x) = (x-1) \cdot \Gamma(x-1)$.

c) Using parts a) and b), compute $\Gamma(2)$, $\Gamma(3)$ and $\Gamma(4)$. What it $\Gamma(n)$ for an arbitrary positive integer n?

#2.
a) Show that the improper integral

$$\int_1^\infty \frac{\ln x}{x^2+1} \, dx$$

converges. (**Hint:** use the fact that for all $x \geq 1$, we have $\ln x \leq x^{1/2}$.

b) Using the method of substitution, show that

$$\int_1^\infty \frac{\ln x}{x^2+1} \, dx \; = \; \int_1^0 \frac{\ln w}{w^2+1} \, dw.$$

(**Hint:** let $w = x^{-1}$).

c) What is

$$\int_0^\infty \frac{\ln x}{x^2+1} \, dx \; ?$$