BE SURE TO SHOW YOUR WORK FOR FULL CREDIT!

NAME:

Scores: (for grader’s use only).

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<td>TOTAL</td>
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1. Evaluate the following integral.

\[ \int \sin^4 x \cos^3 x \, dx \]
2. Evaluate the following integral.

\[
\int_0^{\pi/12} x + 4 \sin(3x) \,dx
\]
3. Evaluate the following integral.

\[ \int_0^1 \tan^3 \theta \sec^3 \theta \, d\theta \]
4. Evaluate the following integral.

\[ \int e^{3x} \sin(4x) \, dx \]
5. Find the area between the graph of

\[ y = \frac{3e^x}{1 + e^{2x}} \]

and the \( x \)-axis between \( x = 0 \) and \( x = e \).
6. A carpet which is 8 meters long is rolled up. When \( x \) meters have been unrolled, a force of \( e^x(64 - x^2) \) Newtons is required to unroll it further. How much work does it take to unroll the entire carpet?
7. Consider the region obtained by rotating the area enclosed by the graphs of \( y = x^2 - 2 \) and \( y = 6 - x^2 \) and \( x = 0 \) about the line \( y = 8 \).

a) Draw a picture of a typical cross-section of the solid at the point \( x = x_i \).

b) What is the area of your cross-section? (Your answer should be in terms of \( x_i \).)
c) Write down a Riemann sum with \( n \) terms which approximates the volume of the solid. Your answer should be in terms of \( \Delta x \) and the \( x_i \).

d) Now write down a Riemann sum only in terms of \( i \) and \( n \) which approximates the volume of the solid. (This time, the expressions \( x_i \) and \( \Delta x \) should not appear in your sum.)
e) Write down an integral representing the exact volume of the solid. You do not need to evaluate the volume.
8. Let \( f(t) \) be the function graphed below. Between \( t = 0 \) and \( t = 4 \), the graph of \( f(t) \) is the same as the graph of \( y = t - 4 \). The function has a global maximum at the point \((5,2)\), and the area of the shaded region is 2. If \( t < 0 \) or \( t > 6 \), then \( f(t) \) is undefined.

\[
y = f(t)
\]

![Graph of y=f(t)](image)

a) Evaluate \( \int_{1}^{6} f(t) \, dt \).

b) Consider the volume formed by rotating the graph of \( f(t) \) around the \( t \)-axis. Let \( h(x) \) be the volume of the part of the solid between \( t = 0 \) and \( t = 2x \). Write down a formula expressing \( h(x) \) as an integral.
c) Write down an expression for $h'(x)$.

d) Evaluate $h(4)$ (not $h'(4)$).
e) Find a number which you know **for a fact** is **larger** than \( h(6) \). Show how you know that your answer is larger than \( h(6) \).
9. Suppose that $f(x)$ is a function with the property that $f(-x) = -f(x)$ for every value of $x$. Show that

$$\int_{-1}^{1} f(x) \, dx = 0.$$ 

**Hint:** Make a substitution.