Answer the following, showing all work clearly and neatly. NOTE: all answers that should be complex numbers should be given in the form $a + bi$ unless stated otherwise.

1. Simplify the following:

   (a) $(3 - i)^2$  
   (b) $i(4 + i)^2$  
   (c) $i^{16} + i^{37} - 7i^{111}$  
   (d) $\frac{x + iy}{x - iy}$

2. Use the quadratic formula to solve the equations, expressing the answers as complex numbers.

   (a) $z^2 + 64 = 0$  
   (b) $2z^2 + 2z + 5 = 0$  
   (c) $5z^2 + 4z + 1 = 0$

3. Prove the following:

   (a) $\frac{z}{w} = \frac{\bar{z}}{\bar{w}}$  
   (b) $\bar{z}^n = z^n$ for any $n \in \mathbb{N}$

4. Compute $\left| \frac{1 + i}{(1 - i)(1 + 3i)} \right|$

5. Describe and sketch the following sets

   (a) $A = \{ z \in \mathbb{C} \mid |z - 3| = 4 \}$  
   (b) $B = \{ z \in \mathbb{C} \mid |z + 3 + i| \leq 2 \}$  
   (c) $C = \{ z \in \mathbb{C} \mid \text{Im}(2z) = 7 \}$  
   (d) $D = \{ z \in \mathbb{C} \mid \text{Re}(3z) < 1 \}$

6. For the circle through the points $0$, $2 + 2i$, and $2 - 2i$, write an equation of the circle using complex numbers, similar to the equation in Example 2 of Section 4.

From the book, §1.5, #8.

From the book, §1.5, #12.