Answer the following, showing all work clearly and neatly. ONLY EXACT VALUES WILL BE ACCEPTED. You may use the result from the bonus problem to help with your calculations even if you do not complete the problem.

1. Parametrize the following descriptions of curves. In other words, for each curve find a function $z(t)$, $t \in [a, b]$ that defines it.

   (a) The line segment from $z_0 = 0$ to $z_1 = e^{i\pi/3}$

   (b) The clockwise circle with center $-2 - i$ and radius 3

   (c) The negatively oriented arc on the unit circle such that $-\pi/4 \leq \text{Arg} \ z \leq \pi/4$

   (d) The positively oriented arc of the circle with center $-3 + 2i$ and radius 5 that starts at $-3 - 3i$ and ends at $2 + 2i$

2. Calculate the following complex integrals.

   (a) $\int_{-1}^{1} f(x) \, dx$ where $f(x) = \begin{cases} e^{i\pi x} & x \in [-1, 0] \\ x & x \in (0, 1) \end{cases}$

   (b) $\int_{1}^{2} \text{Log}(ix) \, dx$

3. Let $C$ denote the line segment from $z = 2$ to $z = 2i$. Notice that for all the points on $C$, the midpoint of the segment is closest to the origin. Without integrating, show that

   \[ \left| \int_{C} \frac{dz}{z^6} \right| \leq \frac{\sqrt{2}}{4} \]

4. Let $C_R$ denote the upper half circle $|z| = R$, with $R \geq 2$ taken in the counterclockwise direction. Show that

   \[ \left| \int_{C_R} \frac{4z^2 - 2}{z^4 + 5z^2 + 6} \, dz \right| \leq \frac{\pi R(4R^2 + 2)}{(R^2 - 2)(R^2 - 3)} \]

   Use this to conclude $\lim_{R \to \infty} \int_{C_R} \frac{4z^2 - 2}{z^4 + 5z^2 + 6} \, dz = 0$, showing all work.

   HINT: use the useful consequence of $\Delta$- inequality: $|\alpha + \beta| \geq \left| |\alpha| - |\beta| \right|$ \quad \forall \alpha, \beta \in \mathbb{C}$.

More problems on the back!
For problems #4 - #8 below, do the following:

(a) Sketch the curve and find a parametrization for the curve.

(b) Calculate the contour integral, showing all work. You may use the result from the Bonus Problem below in your calculations even if you don’t complete the problem.

4. \[ \int_{C_1(0)} (2z + i) \, dz \]

5. \[ \int_{C} \frac{1}{z + 1} \, dz \], where \( C \) is made up of line segments from \(-i\) to \(2 - i\) to \(2 + i\) to \(-i\).

6. \[ \int_{C} \text{Re} z - 2(\text{Im} z)^2 \, dz \], where \( C \) is the same curve as in #5.

7. \[ \int_{\gamma} z \, dz \], where \( \gamma(t) = e^{it}, \, t \in [0, \pi] \)

8. \[ \int_{C} z \, dz \], where \( C \) is the same curve as in #5.

BONUS PROBLEM: (worth up to 4 points)

Prove that for any fixed \( \alpha, \beta \in \mathbb{C} \),

\[ \frac{d}{dt} \log(\alpha t + \beta) = \frac{\alpha}{\alpha t + \beta} \]

as long as \( \alpha t + \beta \) is not on a branch-cut for a branch of \( \log z \) used in calculations.