SHOW ALL WORK. Only exact values will be accepted.

1. Consider \( f(z) = \frac{3}{(1 + z)(2 - z)} \).

   (a) There are three different domains on which a Maclaurin Series and/or Laurent Series for \( f(z) \) expanded about \( z_0 = 0 \) are valid. State them.

   (b) For each of these domains, find the Maclaurin/Laurent Series for \( f(z) \) expanded about \( z_0 = 0 \). HINT: partial fractions and geometric series will be useful.

2. Consider \( f(z) = \frac{e^z - 1}{z^5} \).

   (a) Find a Laurent Series for \( f(z) \) about \( z_0 = 0 \), stating the domain (open annulus) on which the series is valid.

   (b) What are the principal parts of \( f(z) \) expanded about \( z_0 = 0 \)?

   (c) Find \( \text{Res}_{z=0} f(z) \)

   (d) For the unit circle \( C \), use the above to find \( \int_C f(z) \, dz \).

3. Consider \( f(z) = \frac{z^2}{z^2 - 1} \).

   (a) What is the largest domain about \( z_0 = 1 \) on which a Laurent Series for \( f(z) \) expanded about \( z_0 = 1 \) is valid?

   (b) On the above domain, find the Laurent Series for \( f(z) \) expanded about \( z_0 = 1 \). HINT: Use long division and rewrite \( \frac{1}{z + 1} \) into an algebraically equivalent fraction to write \( \frac{1}{z + 1} \) as a geometric series involving powers of \( z - 1 \).

   (c) What are the principal parts of \( f(z) \) expanded about \( z_0 = 1 \)?

   (d) Find \( \text{Res}_{z=1} f(z) \).

   (e) State/give an example of a positively oriented, simple closed curve \( C \) that is within the domain for the Laurent Series. Then use the above to find \( \int_C f(z) \, dz \).
4. Consider \( f(z) = \frac{\sin z}{(z - \pi)^2} \).

(a) Find a Laurent Series for \( f(z) \) about \( z_0 = \pi \), stating the domain on which the series is valid.

(b) What are the principal parts of \( f(z) \) expanded about \( z_0 = \pi \)?

(c) Find \( \text{Res}_{z=\pi} f(z) \).

(d) State/give an example of a positively oriented, simple closed curve \( C \) that is within the domain for the Laurent Series. Then use the above to find \( \int_C f(z) \, dz \).