§5.2, #2: Part (a) for \( f + g \) was done in class. You need to write up the proof for \( f - g \). For part (c), fill in the blanks of the following proof.

**Proof.** Let \( f, g \) be differentiable at \( a \) with \( g(a) \neq 0 \). Then

\[
\left( \frac{f}{g} \right)'(a) = \lim_{x \to a} A
\]

\[
= \lim_{x \to a} B \quad \text{(Hint for part B, just simplify } A \text{ getting rid of double fractions)}
\]

\[
= \lim_{x \to a} C \quad \text{(Hint for part C: add and subtract } f(a)g(a) \text{ in numerator)}
\]

\[
= \lim_{x \to a} \left[ \frac{g(a)}{g(x)g(a)} D - \frac{f(a)}{g(x)g(a)} E \right] \tag{*}
\]

Since \( g \) is differentiable at \( a \), \( g \) is continuous at \( a \) by Theorem [F]. And, since \( g(a) \neq 0 \), we have \( 1/g \) is continuous at \( a \) by Theorem [G]. Thus

\[
\lim_{x \to a} \frac{1}{g(x)} = H.
\]

Since \( f \) and \( g \) are differentiable at \( a \), both

\[
\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{and} \quad \lim_{x \to a} \frac{g(x) - g(a)}{x - a}
\]

exist and equal \( f'(a), g'(a) \), respectively. Thus by using limit laws on (*) , we get

\[
\left( \frac{f}{g} \right)'(a) = I,
\]

which proves the Quotient Rule. \( \square \)