MA 302 Fall 2007  Final Project 1

Due by the 4:00 PM Thursday, December 13

Instructions: You must work on this project entirely on your own. The work you hand in should be presentable and easy to read in the form of a report that summarizes your findings rather than a “question/answer” format. Any calculations or formulas used should be explained, and supported by the graphs. Do not highlight each of the tasks, but you should highlight the different sections (the business or government side). Any graphs you include should be appropriately labeled. Any m-files that you write and/or MATLAB output that shows how your programs work should be included as an Appendix to your report. Your score will depend on overall format (including graphs and such), writing, accuracy and the efficiency/accuracy of the MATLAB programming involved.

1 Introduction

This project deals with an economics application of monopoly pricing. When there is only one company that offers a certain service or product, then it is called a monopoly. Examples include local telephone services, electric and gas companies and cable providers. Monopolists have the advantage over other companies that must compete, since without regulation by local or federal governments, they can potentially control the price of their service or product by controlling the quantity produced. A product that is in short supply will fetch a higher price if the demand for it is high. Conversely, if the product is easy to come by then its price will be low. Monopolies could be detrimental to consumers if the company was not interested in providing enough of their product or service, but rather they were interested in providing just enough to maximize their profits.

Some companies that have received a lot of attention as potential monopolies are cable TV companies. People have been dissatisfied with having to pay extra fees for special cable channels. The question of regulating the cable companies has been a matter of some concern. In this project, you will answer questions about how monopolies set prices and whether or not the cable companies should be regulated by the government.

2 Going into Business

Suppose a small town has offered to give you the rights to provide cable TV service to 200 families that do not have cable TV. As a business person you are interested in maximizing your profit, and your cost of providing cable TV is $20 per month per family plus $3000 in monthly overhead that is related to maintenance of your equipment (regardless of how many families get your service). One hundred families live in houses and one hundred live in apartments. It has been estimated that people living in houses are more likely to order your services, and if you charge a monthly price of $p$ for cable TV then the number $q_h$ of families living in houses who would pay for cable is given
by

\[ q_h(p) = \begin{cases} 
50 & \text{if } p \leq 100 \\
\frac{200 - p}{2} & \text{if } 100 \leq p \leq 200 \\
0 & \text{if } p \geq 200 
\end{cases} \]

The following expression gives the number \( q_a \) of families living in apartments who will pay for hookup if you charge a price of \( \$p \):

\[ q_a(p) = \begin{cases} 
150 - \frac{p}{3} & \text{if } p \leq 150 \\
0 & \text{if } p \geq 150
\end{cases} \]

These are known as demand functions or demand curves. Note that the town giving you the franchise will only allow you to set one price for cable TV, regardless of it being in a house or apartment.

**Task 1:** Create two functions \( q_h \) and \( q_a \) in MATLAB for these demand functions. Each function should take as input either a price \( p \) or a vector of prices and return the computed \( q_h \) or \( q_a \) values for the input. Make a plot of the two demand functions in terms of \( p \) on the same axis, for the price range of \( 0 \leq p \leq 200 \). The axis should be adjusted accordingly to see a complete picture of what is going on, complete with legends, labels and such. As a monopolist your goal is to determine what price \( p \) you should charge for cable TV so that your profit is maximized. This will be done with a series of tasks described below.

**Task 2:** Your revenue is the amount of money that you take in from cable TV in one month. It is equal to the price you charge times the number of cable TV hookups you sell. Express the revenue as a function of \( p \), keeping in mind that the number of sales to houses and apartments is given by \( q_h \) and \( q_a \), respectively. Plot the revenue as a function of \( p \) for the price range \( 0 \leq p \leq 200 \).

**Task 3:** Express your total monthly cost in providing cable service in terms of \( p \). Remember, the costs are \$20 per hookup per month plus \$3000 in overhead costs. Plot the total cost as a function of \( p \) for the price range \( 0 \leq p \leq 200 \).

**Task 4:** You are now (potentially) ready to go into business. The monthly profit from running your cable company is your total revenue from sales minus the total cost to you of providing the service. Express your profit as a function of the price \( p \) you charge for cable and obtain a plot of it for the price range \( 0 \leq p \leq 200 \).

**Task 5:** The plot from Task 4 should indicate what is the optimal price to maximize your profit. Calculate mathematically what the maximum profit is and for what value of \( p \) this maximum occurs (your answer should agree with your graph!)

**Task 6:** At the optimal price you are going to charge for cable, how many families living in houses are paying for your service? How many families living in apartments are paying? Suppose now that the town demands that you pay a monthly fee for this privilege. How much are you willing to pay so that your company can still remain profitable? (There is no one exact answer, since the fee
would have to vary from 0 to the maximum profit you will be making – what would be a reasonable, business-oriented answer?)

3 Going into politics

Suppose now that instead of going into business, you are part of the government of the small town that has given someone the right to provide cable TV. This company has been in business for several years and has led to some complaints. The biggest complaint is that for people living in apartments, the price is so high that they cannot afford it. The company claims that they must charge a high price to remain in business, but suggests that the city government allows the company to charge different prices for apartments and houses.

Task 7: Suppose that the monopolist can charge a price $p_a$ to families living in apartments and a price $p_h$ to families living in houses. What pair of prices maximizes profit? How many houses and apartments are served at these prices? How large of a service fee could the town charge and still allow the monopolist to stay in business?

Note: to answer the questions in Task 7, you will need to maximize a function of two variables. This can be done in a number of ways: multivariable calculus, a MATLAB built-in command, or a graphical approach (which is not as efficient). You are free to choose any of these options, but in any case you must document your approach and explain all of your steps.