There are many different techniques that we’ve learned in order to integrate functions. How can we tell which technique to try first? After calculating MANY integrals, it becomes easier to recognize, but there is a basic strategy.

1. **Definite Integrals.** Check for continuity. Adjust to improper integrals using limits if necessary (§7.8).

2. **Simplify integrand.** (if possible). Usually this involves multiplying factors together or rewriting trigonometric functions using identities.

3. **Look for substitutions.** If you see something that you could substitute \( u = g(x) \) and you also have \( du = g'(x) \, dx \) (give or take a constant coefficient), then the substitution rule may work.

4. **Classify type.** Look to see what the integrand looks like to narrow down the technique used:
   - trigonometric functions (§7.2)
   - rational functions (substitution rule or §7.4)
   - integration by parts: this is used most commonly when you have a product of a polynomial with a transcendental function (trig, exponential, logarithmic) or a combination of seemingly unrelated transcendental functions. Remember, first try to see if substitution rule works. Look at the types of integrands that appear in §7.1 to give you an idea of when it is used.
   - radicals (substitution rule or §7.3)

5. **Try again.** Try a more complicated substitution, integration by parts using a different break-up, or manipulate the integrand. This manipulation usually involves something like multiplying numerator and denominator by some sort of conjugate of the denominator or some sort of “backwards” substitution.

So let’s try a few and see if you can recognize what technique to use.

**Example 1** \[ \int e^{x+x^2} \, dx \] First we would rewrite it as \[ \int e^x e^{x^2} \, dx. \] Then one might try substitution. Let \( u = e^x \). Then \( du = e^x \, dx \). Then we have \[ \int e^x e^{x^2} \, dx = \int e^u du = e^u + C = e^{e^x} + C. \]

**Example 2** \[ \int \frac{\cos x}{\sin x (\sin x - 1)} \, dx. \] First try substitution with \( u = \sin x \), then use partial fractions.
Look at each of the following integrals and determine which technique would need to be used. For example, for example 1, one would write: rewrite integrand as $e^x e^{x^2}$ and then use substitution with $u = e^x$. And for $\int \tan^2 4x \, dx$, one would write that you would convert the $\tan^2 4x$ to $\sec^2 4x - 1$ and then use substitution with $u = 4x$. YOU DO NOT NEED TO COMPLETE THE INTEGRATION (but could you, if you had to?).

1. $\int \frac{\sqrt{9 - x^2}}{x} \, dx$

2. $\int x^3 \ln \sqrt{x} \, dx$

3. $\int \frac{x}{x^2 + 3x + 2} \, dx$

4. $\int_{-1}^{4} x^{-\frac{1}{3}} \, dx$

5. $\int \frac{\cos^3 x}{\cos x \sin^2 x - \cos x} \, dx$

6. $\int \frac{e^{2x}}{1 + e^x} \, dx$