

**Midterm**  
**Partial Differential Equations**  
**Spring, 2009**

Honor code pledge: On this exam I have upheld the ideals of the honor code.

Your name (printed): \_\_\_\_\_

Your signature: \_\_\_\_\_

1. Show that for any nonzero constant  $A$ , the following function  $f(x)$  is a solution of the heat equation  $ku_{xx} = u_t$  (make sure all your steps are clear and follow from previous steps)

$$u(x, t) = \frac{1}{\sqrt{At}} e^{-x^2/(4kt)}$$

2. Find the general solution of the following PDE

$$u_{xt} = x^2 t \sin(xt).$$

3. The function

$$u(x, t) = e^{-\lambda^2 kt} (A \sin(\lambda x) + B \cos(\lambda x))$$

is a nontrivial solution of

$$\begin{aligned} ku_{xx} &= u_t, \quad 0 \leq x \leq L \\ u_x(0, t) &= 0, \quad u(L, t) = 0 \end{aligned}$$

for the right values of  $A$ ,  $B$  and  $\lambda$ . Find the right values for each of  $A$ ,  $B$  and  $\lambda$ : justify your assertions.

4. State the general solution of each of the following PDE's and BC's, including  $\lambda$ , (no justification is needed)

(a)  $ku_{xx} = u_t, 0 \leq x \leq L$   
 $u_x(0, t) = 0, u_x(L, t) = 0$

(b)  $ku_{xx} = u_t, 0 \leq x \leq L$   
 $u(0, t) = 0, u_x(L, t) = 0$

(c)  $ku_{xx} = u_t, 0 \leq x \leq L$   
 $u(0, t) = 0, u(L, t) = 0$

5. Find a simplified formula for the Fourier coefficient  $a_n$  (i.e. the cosine coefficient),  $n \geq 1$ , for the function  $f(x) = -x^2 + 1$  on the interval  $[-1, 1]$

(Hint: you may want to use integration by parts  $\int f' \cdot g = f \cdot g - \int f \cdot g'$  or  $\int u dv = uv - \int v du$ ;

also, you should have the formula for  $\int x \sin(ax) dx$  and  $\int x \cos(ax)$  on your note card, you can use this to save some work; finally, double check your anti-derivative).