7.4 Integration of Rational Functions

Example 1. Find \( \int \frac{1}{x^2 + a^2} \, dx \).

Example 2. Verify the fact that \( \frac{1}{x^2 - a^2} = \frac{1/(2a)}{x - a} - \frac{1/(2a)}{x + a} \) and use this to find \( \int \frac{1}{x^2 - a^2} \, dx \).
Example 3. Find 
\[ \int \frac{2x + 3}{x^2 - 9} \, dx \]

Example 4. (a) Use partial fractions to split the fraction \( \frac{-3x - 7}{x^2 + 7x + 12} \) into two fractions.
(b) Verify that your partial fraction solution works, by combining your answers with a common denominator.
(c) Find \[ \int -\frac{3x + 7}{x^2 + 7x + 12} \, dx. \]
Example 5. [Stewart, 6e, #4b] Set up the partial fraction expansion for
\[
\frac{2x + 1}{(x + 1)^3(x^2 + 4)^2}
\]

Example 6. [Stewart, 6e, #17] Find
\[
\int_1^2 \frac{4y^2 - 7y - 12}{y(y + 2)(y - 3)} \, dy
\]
Example 7. Find \( \int \frac{x^2 + 3x + 1}{x^3 + x} \, dx \).
Example 8. Use polynomial division to find \( \int \frac{x^4 + x^3 - 2x^2 + 17x + 2}{x^2 + 1} \, dx \).

Example 9. Use completing the square to find \( \int \frac{1}{x^2 + 6x + 7} \, dx \).
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Note that $x^2 + 6x + 9 = (x + 3)^2$, and simplify $9 + 7$ to $2$ to get $x^2 + 6x + 7 = (x + 3)^2$

Now we can finish the integral

$$
\int_1 \frac{2x^3 - 3x + 7}{x + 1} \, dx
$$

Extra examples

Example 10. Find $\int \frac{2x^3 - 3x + 7}{x + 1} \, dx$

Example 11. Find $\int \frac{x}{x^2 + 4x + 10} \, dx$
Example 12. Apply partial fractions to split up \( \frac{x}{(x^2 + 2x + 2)(x^2 - x + 3)} \).
Example 13. Set up (but do not solve) the following as a partial fraction:

\[ \frac{x^4 + 3x^2 - 17x + 11}{x(x + 1)^2(x^2 + 2x - 7)} \]

Example 14. Set up (but do not solve) the following as a partial fraction:

\[ \frac{x^7 - 1324x^2 + x - 10}{(x^2 + 1)(x^2 + 10)^4} \]